水文学基礎 Fundamentals of Hydrology (蒸発散 3 Evapotranspiration 3) HW

1. The temperature T, the horizontal wind velocity u, the vapor pressure e and the atmospheric pressure p were measured at the same altitude z. The water surface temperature T_s was also measured. Derive equations to calculate the water vapor flux and the sensible heat flux from the water surface, using the following equations
$$E = -\frac{K_E}{K_M} \frac{k^2 \rho (q_4 - q_3)(u_2 - u_1)}{\ln(z_2/z_1)\ln(z_4/z_3)} \tag{1}$$
and
$$H = -\frac{K_H}{K_M} \frac{k^2 \rho C_p (T_4 - T_3)(u_2 - u_1)}{\ln(z_2/z_1)\ln(z_4/z_3)} \tag{2}$$
for estimating the water vapor flux and the sensible heat flux. Note that the height above the water surface where the wind velocity is 0 is z_0, that the height where the specific humidity equals the saturated specific humidity $q_s(T_s)$ on the water surface is z_{0E}, and that the height where the temperature equals of the water surface temperature T_s is z_{0H}.

2. At 2m, temperature was 25°C, relative humidity was 40 %, atmospheric pressure was 1013 hPa, wind velocity was 3 m·s^{-1} and temperature on the water surface was 25°C. $z_0 = z_{0E} = z_{0H} = 0.03$ cm is supposed, and $k = 0.4$, at temperature of 25°C the air density $\rho = 1.19$ kg·m^{-3}. Under the assumption of neutral atmospheric condition, caluculate water vapor flux with unit of kg·m^{-2}s^{-1}. Suppose that at temperature of 25°C the water density is 997 kg·m^{-3}, caluculate water vapor flux with unit of mm·day^{-1}.

Answer:

Substituting $q_4 = q$, $z_4 = z$, $q_3 = q_s(T_s)$, $z_3 = z_{0E}$, $u_2 = u$, $z_2 = z$, $u_1 = 0$, $z_1 = z_0$ into Eq. (1) and using $q \simeq 0.622e/p$,
$$E = -\frac{K_E}{K_M} \frac{0.622 k^2 \rho (e - e_s(T_s)) u}{p \ln(z/z_0)\ln(z/z_{0E})} \tag{3}$$
Substituting $T_4 = T$, $z_4 = z$, $T_3 = T_s$, $z_3 = z_{0H}$, $u_2 = u$, $z_2 = z$, $u_1 = 0$, $z_1 = z_0$ into Eq. (2),
$$H = -\frac{K_H}{K_M} \frac{k^2 \rho C_p (T - T_s) u}{\ln(z/z_0)\ln(z/z_{0H})}$$

At temperature of 25°C the saturation vapor pressure $e_s = 6.1078 \times 10^{7.5\times25/(237.3+25)} = 31.675$ hPa. Substituting observed values into Eq. (3),
$$\begin{aligned}
E &= -\frac{0.622 k^2 \rho (e - e_s(T_s)) u}{p \ln(z/z_0)\ln(z/z_{0E})} \\
&= -\frac{0.622 \times 0.4^2 \times 1.19 \times (31.675 \times 0.4 - 31.675) \times 3}{1013 \times \ln(2/0.0003) \times \ln(2/0.0003)} \text{ kg} \cdot \text{m}^{-2}\text{s}^{-1} \\
&= 8.598 \times 10^{-5} \text{ kg} \cdot \text{m}^{-2}\text{s}^{-1} \\
&= 8.598 \times 10^{-5} \times 997 \times 1000 \times 87600 \text{ mm} \cdot \text{day}^{-1} \\
&\approx 7.45 \text{ mm} \cdot \text{day}^{-1}
\end{aligned}$$

Hints:

- Suppose in equations (1) and (2), $u_1 = 0$ at $z_1 = z_0$ and T, u and e were measured at $z_4 = z_2 = z$.
- Suppose that q_3 is $q_s(T_s)$ at $z_3 = z_{0E}$ in eq (1) and that T_3 is T_s at $z_3 = z_{0H}$ in eq (2).
- The specific humidity q can be approximated using the vapor pressure e and atmospheric pressure p as $q \simeq 0.622e/p$.

- In neutral atmospheric condition, $K_M = K_E = K_H$.
- $e_s(T)$ which is the saturated vapor pressure over the temperature T is useful. This can be expressed as $e_s(T) = 6.1078 \times 10^{aT/(b+T)}$, Here, a = 7.5, b = 237.3, unit of e_s is hPa and unit of T is °C. As a result, the vapor pressure e can be derived by multiplying the saturated vapor pressure and relative humidity.