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SCALE INVARIANT TOPMODEL FOR THE SOLUTION OF DEM RESOLUTION EFFECTS AND PARAMETER INCONSISTENCY IN HYDROLOGICAL MODELING

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The topographic index of TOPMODEL is scale dependent which leads identified parameter values to be dependent on a DEM resolution. This makes difficult to use model parameter values identified with a different resolution TOPMODEL. To overcome this problem, this research has developed a concept of resolution factor to account for the scale effect in up-slope contributing area per unit contour length in the topographic index and a fractal method for scaled steepest slope as an approach to account for the scale effect on slopes. The method has been applied to Kamishiiba catchment (210 km^2) in Japan and it is shown that the downscaled topographic index distribution is similar to a target resolution DEM topographic index distribution. The method to downscale the topographic index distribution is then coupled with the TOPMODEL to develop the Scale Invariant TOPMODEL and is applied in Kamishiiba catchment (210 km^2). It is shown that the simulated runoff from the Scale Invariant TOPMODEL applied at 1000m grid resolution DEM, with the same set of effective parameter values derived from 50m grid resolution DEM, have matched with the simulated runoff of the 50m DEM resolution TOPMODEL. It is also shown that the simulated runoff from the Scale Invariant TOPMODEL applied at 1000m grid resolution DEM, with the same set of effective parameter values derived from 50m grid resolution DEM, have matched with the observed runoff with high efficiency without recalibration.

INTRODUCTION

In the field of hydrology, since the introduction of the first blueprint of a distributed hydrological model (Freeze and Harlan [1]) the desire to develop more physically realistic distributed models has been motivated for forecasting changes in hydrological behavior due to a variety of land use and climate changes and for hydrologic predictions in ungauged basins.

The fact that a model may be physically based in theory but not consistent with observations results primarily from the mismatch in scales between the scale of observable state variables and the scale of applications. Conceptualizations of the homogeneity of the hydrological quantities inside the grid of a DEM result in different performances in the models themselves with variations in the assumed scale.

Several researches (Wolock and Price [2]; Zhang and Montgomery [3]) have discussed the effects of digital elevation model map scale and data resolution on the distribution of the topographic index, concluding that there is interdependence between DEM scale and topographic index distribution. Band *et al.* [4] point out that higher frequency topographic information is lost as the larger sampling dimensions of the grids act as filter. Lack of a translation method of the scale dependence relation into effective hydrological models have posed a serious problem for the ungauged basins of developing countries where only coarse resolution DEM data is available (Pradhan *et al.* [5]).

In this study we focus on the influence of DEM resolution on slope angle, upslope contributing area, and develop a method to downscale the topographic index of TOPMODEL by incorporating scaling laws. By using the method, the topographic index distribution of fine resolution DEM is successfully derived by using only coarse resolution DEM (Pradhan *et al.* [6]). Then we develop a Scale Invariant TOPMODEL by coupling the method to down scale the topographic index distribution with TOPMODEL and it is shown that the Scale Invariant TOPMODEL is consistent with observation when applied at coarse resolution DEM with the parameter identified at fine resolution DEM.

DEM RESOLUTION EFFECTS ON TOPOGRAPHIC INDEX DISTRIBUTION

Topographic index of TOPMODEL is defined as

$$TI = \ln\left\{\frac{a}{(\tan\beta)}\right\} \tag{1}$$

where a is the local up-slope catchment area per unit contour length and β is the slope angle of the ground surface. TOPMODEL allows for spatial heterogeneity by making calculations on the basis of the topographic index distribution. Topographic index being scale dependent, leads identified parameter values to be dependent on a DEM resolution.

In Figure 1, topographic index distribution is shifted towards higher value, as DEM resolution gets coarser in the Kamishiiba catchment (210 km^2) in Japan. Table 1 shows DEM resolution effect on a spatial mean value of the topographic index λ in Eq. (2).

$$\lambda = \frac{1}{A}\int_A \ln(a_i/\tan\beta_i)\,dA \tag{2}$$

Table 1 DEM resolution effect on topographic constant, λ, in Kamishiiba catchment.

DEM Resolution (m)	**50**	**150**	**450**	**600**	**1000**
Topographic constant λ [ln(m2)]	**6.076**	**7.423**	**9.222**	**9.622**	**10.353**

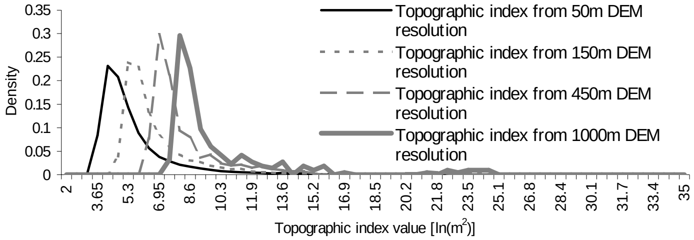

Figure 1. Effect of DEM resolution on density distribution of topographic index.

Considering lateral transitivity to be constant in a subcatchment or catchment, then the key role for hydrological similar condition is played by the distribution function of topographic index. Higher frequency topographic information contained in topographic index is lost as the larger sampling dimensions of the grids act as filter. This makes the hydrological similarity condition accounting combined soil-topographic index, to vary with the variation in DEM resolution used. To overcome this problem, this research has developed a method to downscale topographic index.

THE METHOD TO DOWNSCALE THE TOPOGRAPHIC INDEX

Resolution factor in topographic index

The smallest contributing area derived from a DEM resolution is a single grid of the DEM at that resolution. Thus area smaller than this grid resolution is completely lost which makes the topographic index distribution from coarse resolution DEM to swift towards higher values (see Figure 1). But as we use finer resolution DEM, the smaller contributing area that is the area of finer grid resolution is achieved. From this point of view we introduced Resolution Factor R_f concept in topographic index as shown in Eq. (3), for detail explanation refer Pradhan *et al.* [6].

$$TI = \ln\left\{\frac{C_i}{(W_i R_f \tan \beta_i)}\right\} \tag{3}$$

where C_i and W_i is the upslope contributing area and unit contour length at a point *i* of the coarse resolution DEM. Taking W^*_i as the unit contour length of the target resolution DEM, Resolution Factor R_f is defined as

$$R_f = \frac{Coarse \quad DEM \quad \mathrm{Re}solution}{T\arg et \quad DEM \quad \mathrm{Re}solution} = \frac{W_i}{W^*_i} \tag{4}$$

Fractal method for scaled steepest slope

Slope derived from coarse resolution DEM is underestimated. To scale the local slope, we followed the fractal theory in topography and slope proposed by Klinkenberg and

Goodchild [9] and Zhang *et al.* [10]. Considering the significant role of the steepest descend slope in hydrological modeling, we developed a modified fractal method to account for the DEM resolution effects on the steepest slope, which is shown in Eq. (5).

$$S_{scaled} = \alpha_{steepest}\, d_{scaled}^{\,(1-D)} \tag{5}$$

where S_{scaled} is the scaled steepest slope from a caorse resolution DEM to the target resolution DEM. d_{scaled} is the scaled steepest slope distance of the target resolution DEM. The direction of the scaled steepest slope in the target resolution DEM is taken as the same direction of the steepest slope in the coarse resolution DEM. It is found that standard deviation of elevation in the same sub area is quite stable for change in DEM resolution. Thus fractal dimension D in Eq. (5) is related to the standard deviation of elevation σ in 3 x 3 moving window pixels as per Zhang *et al.* [10] as shown by Eq. (6).

$$D = 1.13589 + 0.08452 \ln \sigma \tag{6}$$

The parameter α is a coefficient in the fractal method for slope proposed by Zhang *et al.* [10] and its values is found to fluctuate very high from one local place to another in comparison to D value. Unlike the method by Zhang *et al.* [10] we developed a new method in which α values are derived directly from the steepest slope of the available coarse resolution DEM, $\alpha_{steepest}$ in Eq. (6), keeping the fact that steepest slope itself represents the extreme fluctuation, for details refer Pradhan *et al.* [6].

SCALE INVARIANT TOPMODEL

By combining Eq. (3) and Eq. (5), the method to downscale topographic index which includes resolution factor to account for the effect of scale in up slope contributing area per unit contour length and a fractal method for scaled steepest slope as an approach to account for the effect of scale on slope is given by Eq. (7).

$$(TI)_{scaled} = \ln\left[\frac{C_i}{\{W_i R_f (\tan \beta_i)_F\}}\right] \tag{7}$$

where, $(TI)_{scaled}$ is the scaled topographic index and $(\tan\beta_i)_F$ is S_{scaled} of Eq. (5) which is the scaled steepest slope by fractal method. The method to downscale the topographic index is combined with TOPMODEL to develop the Scale Invariant TOPMODEL.

RESULTS AND DISCUSSION

The method to downscale the topographic index and the Scale Invariant TOPMODEL is applied to Kamishiiba catchment (210 km^2), Japan.

Table 2 Topographic constant λ value for scaled DEM from 1000 m grid resolution to finer grid resolutions in Kamishiiba catchment.

Topographic constant, **λ** [ln(m^2)], value for scaled DEM from 1000 m grid resolution to			
50 m target grid resolution	150 m target grid resolution	450 m target grid resolution	600 m target grid resolution
6.474	7.573	9.11	9.604

Application of the method to downscale the topographic index distribution

The down Scaled values of λ in Eq. (2) from 1000m grid resolution DEM to finer DEM resolutions in Table 2 are almost equal to the λ values in Table 1 from the fine grid resolution DEMs.

Figure 2 shows the perfect fit of density function of scaled topographic index distribution from 1000m grid resolution DEM to various grid resolution DEMs by using scale invariant model. It is found that between 50m-grid resolution DEM and 150m-grid resolution DEM where the slope obtained is more precise and does not vary significantly, resolution factor R_f alone played the dominant role in the scale invariant model.

Figure 3a is the topographic index distribution using 1000m DEM. Figures 3 b, c, d and e are the scaled topographic index distribution obtained by using the scale invariant model with the same 1000m grid resolution DEM to 600m, 450m, 150m and 50m grid resolution DEM respectively. Figure 3f is the topographic index distribution using 50m DEM. Distinct difference can be seen between spatial distribution of the topographic index in Figure 3a and Figure 3f that are from 1000m-grid resolution DEM and 50m-grid

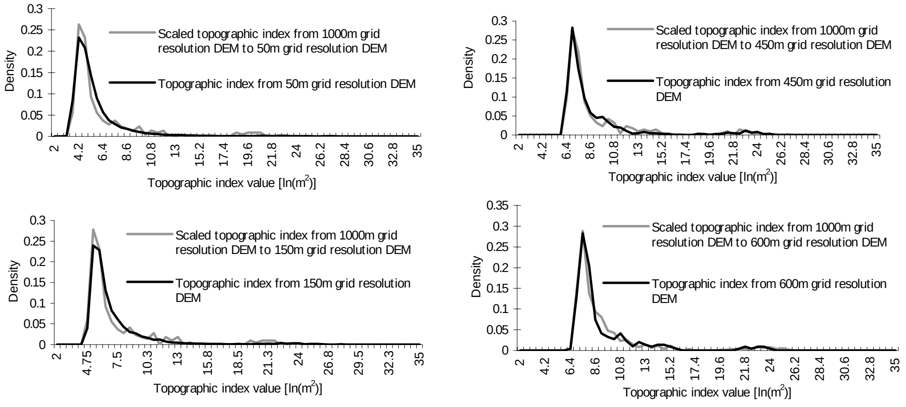

Figure 2. Comparison of scaled topographic index distribution from 1000m DEM resolution to finer grid resolution DEM and at the fine scale in Kamishiiba catchment.

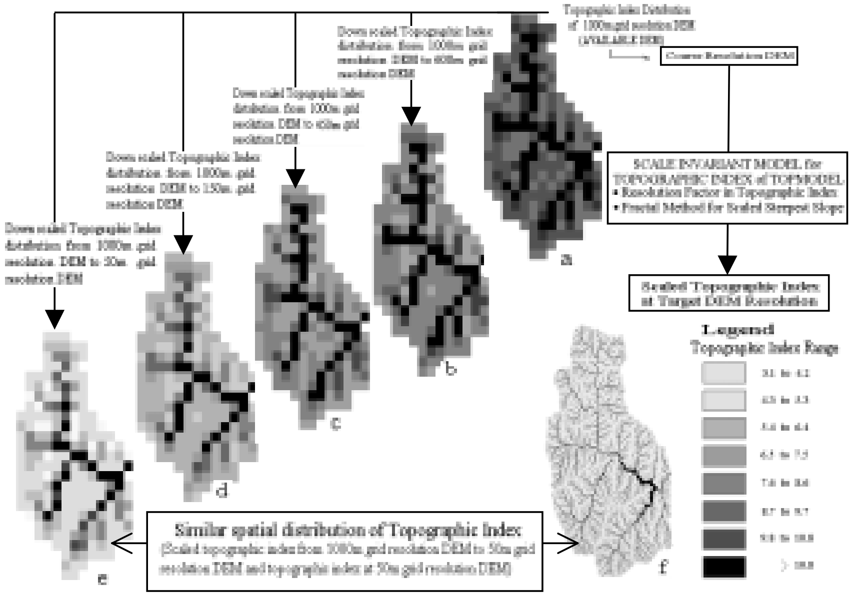

Figure 3. Spatial distribution of scaled topographic index applied to Kamishiiba catchment (210 km^2). (a) topographic index distribution using 1000m DEM resolution, (b) scaled topographic index distribution obtained from 1000m DEM resolution to 600m DEM resolution, (c) scaled topographic index distribution from 1000m DEM resolution to 450m DEM resolution, (d) scaled topographic index distribution from 1000m DEM resolution to 150m DEM resolution, (e) scaled topographic index distribution from 1000m DEM resolution to 50m DEM resolution, (f) topographic index distribution using 50m DEM resolution.

resolution DEM respectively. The spatial distribution of topographic index displayed by Figure 3e has matched the existing reality displayed by Figure 3f.

Application of Scale Invariant TOPMODEL

Figure 4a and 4b shows simulation results by TOPMODEL (with and without coupling the downscaling method of topographic index) in Kamishiiba catchment (210 km^2) for the same rainfall event. For all the simulation results indicated in Figure 4, the used effective parameter of the TOPMODEL are identified by 50m DEM resolution TOPMODEL (see Table 3) which gave the Nash efficiency of 94%. When applying the same parameter values to 1000m DEM resolution TOPMODEL, Nash efficiency tremendously dropped down to negative value, -45% (see Figure 4a). Figure 4a shows completely different and erroneous performance of 1000m DEM resolution TOPMODEL

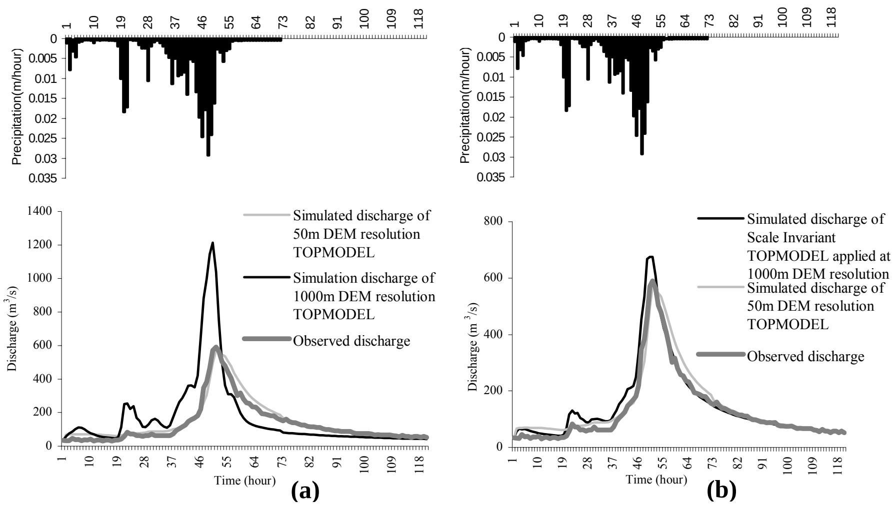

Figure 4. Simulation results of TOPMODEL (with and without coupling the downscaling method of topographic index) applied to Kamishiiba catchment (210 km^2). (a) Completely different and erroneous performance of the 1000m DEM resolution TOPMODEL from that of the 50m DEM resolution TOPMODEL when applying the same set of effective parameter values calibrated at the 50m DEM resolution. (b) Perfect match of the simulated discharge of the Scale Invariant TOPMODEL with that of the simulated discharge of 50m DEM resolution TOPMODEL and the observed discharge under the same effective parameter values calibrated at the 50m DEM resolution.

from that of the 50m DEM resolution TOPMODEL when applying the same set of effective parameter values calibrated at the 50m DEM resolution. This blunder is the consequence of the dependence of the topographic index distribution on a DEM resolution, which is shown earlier in Figure 1 and Table 1. Figure 4b shows perfect match of the simulated discharge of the Scale Invariant TOPMODEL with that of the simulated discharge of 50m DEM resolution TOPMODEL and the observed discharge under the same effective parameter values calibrated at the 50m DEM resolution, Nash Efficiency obtained is 90%.

Table 3 Effective parameter values identified by 50m DEM resolution TOPMODEL.

Lateral transmissivity of soil at saturation condition *To* (m^2/hr)	Decay factor of lateral transmissivity with respect to saturation deficit *m* (m)	Maximum root zone storage Rzmax (m)
9.8	0.07	0.001

CONCLUSION

This research has developed a Scale Invariant TOPMODEL, which is independent of DEM resolution effects and consistent in parameter values with observations although scale of observable state variables and scale of application are mismatched. Analyzing the scale laws this research has developed concept of resolution factor to account for the effect of scale in up slope contributing area per unit contour length in topographic index and a fractal method for scaled steepest slope as an approach to account for the effect of scale on slopes, which are combined to develop the method to downscale topographic index distribution. The method to downscale the topographic index is then coupled with TOPMODEL to develop the Scale Invariant TOPMODEL. It is hoped that the findings of this research seeks its applicability as a tool to a wider range of boundary as per the scale problems in hydrological processes and solution approach is concerned.

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