SPATIAL RAINFALL FIELD SIMULATION WITH RANDOM CASCADE INTRODUCING OROGRAPHIC EFFECTS ON RAINFAL

Yasuto TACHIKAWA Disaster Prevention Research Institute, Kyoto University Gokasho, Uji, 611-0011, Japan

Masahiro HIWASA

Graduate School of Civil Engineering, Kyoto University Yoshida Honmachi, Sakyo-ku, Kyoto, 606-8501, Japan

Kaoru TAKARA Disaster Prevention Research Institute, Kyoto University Gokasho, Uji, 611-0011, Japan

A space rainfall model is constructed to generate spatial rainfall fields. The model is based on a random cascade process which disaggregates a regionally averaged rainfall amount to generate spatial distributions of rainfall. To include a structure of rainfall field heterogeneity in rainfall simulations, a deterministic factor is introduced in a random cascade generator. The factor represents an orographic effect on spatial distributions of rainfall, which is determined by using the dependence line on topographic elevations (DLTE) developed by Nakakita *et al.* [1] and Suzuki *et al.* [2]. The model is applied to a 192 km \times 192 km area covered by the Miyama radar rain gauge in the Kinki region of Japan. It is found that simulation results well preserve the characteristics of orographic effects represented by the DLTE. This suggests that the proposed method has the ability to capture statistical characteristics of topography on rainfall fields found in radar rainfall data.

INTRODUCTION

A downscaling technique to generate local scale rainfall distributions from global scale information is newly devised for local scale watershed managements in ungauged basins. In the method, a statistical downscale technique using a random cascade theory considering orographic effects on rainfall distributions is proposed.

The WGR model based on the point process theory developed by Waymire *et al.* [3] is a well-known stochastic model to generate a space-time rainfall distribution; however it is quite difficult to identify the model parameters. On the other hand, random cascade models, which assume self-similarity in spatial rainfall distributions, have recently used as a rainfall field generator, for example, see Over and Gupta [4]. These models have few model parameters which are easily obtained using observed spatially distributed rainfall information. To represent non-homogeneous spatial features of rainfall patterns,

Jothityangkoon *et al.* [5] introduced the regional characteristics of rainfall into the cascade generator with the beta-lognormal model developed by Over and Gupta [4].

In this study, a topographic effect on rainfall spatial distributions is incorporated into a random cascade generator to realize more realistic rainfall field simulations. It is known that a spatial distribution of accumulated rainfall is well related to a topographic elevation pattern. To include the topographic effects into simulated rainfall distributions, the dependence line on topographic elevations (DLTE), which represents the dependence properties of spatial rainfall distributions on topographic elevations developed by Nakakita *et al.* [1] and Suzuki *et al.* [2], is incorporated into a random cascade generator. DLTE is a regression line between the spatial averages of accumulated rainfall and topographic elevations. Nakakita *et al.* [1] and Suzuki *et al.* [2] found that the correlation coefficient of the relationship obtained using radar rainfall data was more than 0.9. Using the DLTE, deterministic topographic multiplier matrices are introduced into a random cascade generator using the beta-lognormal model.

The method is applied to a 192 km \times 192 km region in the Kinki area of Japan. The parameters of the random cascade generator are obtained using radar rainfall data. The values of the deterministic topographic multiplier matrices for each downscale level are determined with the DLTE and average elevation of each sub-area. By using the cascade generator, spatial rainfall distributions with 3km resolution are simulated over the region. Then statistical characteristics of the generated data are examined. It is found that the accumulated values of generated rainfall fields with 3 km grid resolution shows a linear relationship with the elevations, which is in good agreement with the relationship introduced by the DLTE. This suggests that the proposed method has the ability to capture statistical characteristics of topography on rainfall fields found in radar rainfall data.

RANDOM CASCADE MODEL

A random cascade process starts spatial disaggregation of rainfall with a given volume of rainfall in an initial area (level 0). The area is subsequently subdivided into sub-areas (four areas in this case) at each disaggregation step and the volume given at the previous step is assigned into the four sub-areas. The rainfall volume in the sub-area i at the n th level of disaggregation step is given by

$$\mu_n^i = R_0 L_0 b^{-n} \prod_{j=1}^n W_j^i$$
(1)

where R_0 is an average rainfall intensity (mm/h) of the initial area, L_0 is the length of the study area (in this case 192 km), *b* is a branching number ($b = 2^d$), and *d* is a dimension constant (d = 2). The multipliers W_j^i are random cascade generators which are weights to assign average rainfall intensity to four sub-areas. As the cascade generator W_j^i , we adopt the beta-lognormal model proposed by Over and Gupta [4]. In the model, *W* is a product of independent random variables *B* and *Y*, that is *W*=*BY*. *B* and *Y* are represented as

$$B = 0 \text{ with probability } P(B = 0) = 1 - b^{-\beta}, B = b^{-\beta} \text{ with probability } P(B = b^{-\beta}) = b^{-\beta}$$
(2)

$$Y = b^{-(\sigma^2 \ln b)/2 + \sigma X}$$
(3)

where *X* is a standard normal random variate and β and σ^2 are parameters of the betalognormal model. The expectation and variance of *B* and *Y* are probed to be

$$E[B] = 1, \quad Var[B] = b^{\beta} - 1$$
 (4)

$$E[Y] = 1, \quad Var[Y] = e^{(\sigma \ln b)^2} - 1$$
(5)

From the Eqs. (4) and (5), E[W] = 1 is satisfied. The random variate *B* expresses the condition whether it rains in the sub-area or not, and the random variate *Y* represents the weight for assigning a rainfall volume into the four sub-areas. Figure 1 shows an example of generated rainfall fields at each disaggregation step (from step1 to step 6) by using the beta-lognormal model. The study area is a 192 km × 192 km area covered by the Miyama radar rain gauge in the Kinki region of Japan.



Figure 1. An example of disaggregated rainfall fields from level 1 to level 6 using the beta-lognormal model applied at a 192 km \times 192 km area covered by the Miyama radar rain gauge in the Kinki region of Japan.

PARAMETER ESTIMATION OF BETA-LOGNORMAL MODEL

The values of model parameters β and σ^2 are identified using the data observed at the Miyama radar rain gauge. The observation range is the inside of the radar radius 120 km.

The study area is a 192 km \times 192 km square area within the radar observation range. The space resolution of the radar rainfall data is 3 km. Thus, the study area is composed of a 64 \times 64 grid system. The time resolution is 5 minute. Radar data of 1000 observations during August 1989 to September 1989 were used to identify the values of model parameters.



Figure 2. Estimated model parameters of β (a) and σ^2 (b) as functions of average rainfall intensity *R*.

The method to identify the parameters is the same as shown in Jothityangkoon *et al.* [5]. In Figure 2, 1000 values of obtained parameters are plotted as a function of spatial average rainfall intensity R (mm/h). The solid lines in the figures are regression curves derived by a least square method. The empirical functions of the regression curves adopted here are the same function type used by Jothityangkoon *et al.* [5].

It is found that when the value of average rainfall intensity *R* increasing, the value of β decreases. This implies when the regional average of rainfall intensity is higher, the possibility that rainfall occurs at somewhere in the area becomes large. For the relationship between σ^2 and *R*, it shows positive correlation when the value of *R* is less than several mm/hr. This shows when the regional average of rainfall intensity is higher in the extent, the variance of *Y* becomes large, which leads to large variations of spatial rainfall intensity. These relationships are similar to the results obtained by Over and Gupta [4] and Jothityangkoon *et al.* [5].

INTRODUCING DEPENDENCE PROPERTIES OF SPATIAL RAINFALL DISTRIBUTION ON TOPOGRAPHY INTO CASCADE GENERATOR

To incorporate a structure of rainfall field heterogeneity in rainfall simulations, a deterministic factor is introduced in a random cascade generator. The factor represents an orographic effect on spatial distributions of rainfall, which is determined by using the

dependence line on topographic elevations (DLTE) developed by Nakakita *et al.* [1] and Suzuki *et al.* [2]. Nakakita *et al.* [1] and Suzuki *et al.* [2] presented an obvious dependence relation between accumulated rainfall distributions and topographic elevations. They examined the relationship between the spatial mean values of accumulated rainfall obtained in the classes divided by 100 m interval of topographic elevations and their mean topographic elevations. They found a high value of the correlation coefficient over 0.9 between the logarithmic values of average accumulated rainfall amount in each class and their mean topographic elevations. The relation (the Dependence Line on Topographic Elevation, DLTE) is represented as

$$\log\left(\mu_k(T)/\mu(T)\right) = a \times z_k + b \tag{6}$$

where $\mu(T)$ (mm) is an accumulated value of spatially averaged rainfall over the entire study region during the accumulation time scale *T*; $\mu_k(T)$ (mm) is the spatial average value of accumulated rainfall amount in the elevation class *k*; z_k (m) is the spatial average of topographic elevation in the elevation class *k*; and *a*, *b* are parameters. Suzuki *et al.* [5] obtained the DLTEs for various $\mu(T)$ in the study area by using the radar rainfall data. When $\mu(T) = 300$ (mm) and setting $y = \mu_k(T)/\mu(T)$, the DLTE is obtained as

$$\log y = 0.00027175 \quad z + \log 0.83099 \tag{7}$$

By using this relationship, deterministic topographic multiplier matrices G are introduced into a random cascade generator. For the subdivision area i ($i = 1, 2, ..., 2^{2n}$) at the disaggregation level n (n=1, 2, ..., 6), the topographical weight G_n^i is incorporated according to Jothityangkoon *et al.* [5] as

$$W = BYG \tag{8}$$

with the condition that the average value of G over the sub-areas is equal to 1 at each disaggregation step. G_n^{i} is given by

$$G_{n}^{i} = \mu_{n}^{i} / \overline{\mu_{n}^{i}}, \quad \mu_{n}^{i} = e^{y_{n}^{i}} \mu(T), \quad \overline{\mu_{n}^{i}} = \sum_{i=1}^{2^{2n}} \frac{e^{y_{n}^{i}} \mu(T)}{2^{2n}}$$
(9)

where y_n^i is the value of y calculated using Eq. (7) at the *n* th level of disaggregation in the *i* th sub-area. Thus the values of the topographic multiplier matrices *G* are determined by the DLTE and topographic elevation information.

Figure 3 shows simulated rainfall fields for the 192 km \times 192 km region with spatial resolution of 3 km at the sixth level of disaggregation using the cascade generator in Eq. (8) for the rainfall field at 12:40 on September 5 in 1989.



Figure 3. Simulated rainfall fields for the 192 km \times 192 km region with spatial resolution of 3 km at the sixth level of disaggregation using the cascade generator introducing topographic effects for the rainfall field at 12:40 on September 5 in 1989.

To examine the effect of the topographic multiplier matrices G, 100 realizations of rainfall distributions were simulated. In the simulations, the initial rainfall intensity for the 192 km × 192 km area at the level 0 was set to 1 (mm/h). Figure 4 shows the relation between the accumulated rainfall volume and the elevation using the simulated rainfall at the sixth level (spatial resolution 3km). The figures (a) and (b) are the cases of 50 and 100 times accumulations respectively. The solid line represents the DLTE set in the simulations.

The number of times of accumulation increases, the more clear correlation relationship between topographic elevations and the amounts of accumulated rainfall appears. This shows the topographic effects on spatial rainfall distributions described by using the DLTE are well incorporated into the random cascade rainfall simulations through the deterministic topographic multiplier matrices G.

However, most of results in Figure 4 are plotted over the DLTE and the gradient calculated using the plotted points is steeper than the one of the DLTE. In the simulations presented in Figure 4, *G* was incorporated at all disaggregation levels. For the reason, the topographic dependence on rainfall distributions appears strongly.

To avoid the overdependence, the simulations that G was incorporated only at the 6th disaggregation level were carried out. Figure 5 shows the results of the simulations. In both cases of 50 times and 100 times accumulations, the results are well plotted around the DLTE, which are better results than shown in Figure 4.



Figure 4. The relation between the accumulated rainfall volume and the elevation using the simulated rainfall at the sixth level (spatial resolution 3km). In the cases, G is incorporated at all disaggregation levels. The figures (a) and (b) are the cases of 50 and 100 times accumulations respectively. The solid line represents the DLTE set in the simulations.

Figure 5. The relation between the accumulated rainfall volume and the elevation using the simulated rainfall at the sixth level (spatial resolution 3km). In the cases, G is incorporated only at the 6th disaggregation level. The figures (a) and (b) are the cases of 50 and 100 times accumulations respectively. The solid line represents the DLTE set in the simulations.

CONCLUSION

In this study, a topographic effect on rainfall spatial distributions was incorporated into a random cascade generator to realize more realistic rainfall field simulations. To include the topographic effects into generated rainfall distributions, the dependence line on topographic elevations (DLTE) of rainfall distributions developed by Nakakita *et al.* [1] and Suzuki *et al.* [2] was incorporated into the random cascade generator. Using the DLTE, deterministic topographic multiplier matrices were introduced into a random cascade generator using the beta-lognormal model.

The method was applied to a 192 km × 192 km region in the Kinki area of Japan. The parameters of the random cascade generator were obtained using radar rainfall data. By using the cascade generator, spatial rainfall distributions with 3 km resolution were simulated over the region. Two kinds of random cascade generators were tested; one is a generator incorporating the deterministic topographic multiplier matrices *G* at every disaggregation step, and another is the one with introducing *G* only at the 6th level of disaggregation. Then statistical characteristics of the simulated rainfall data were examined. It was found that the accumulated values of generated rainfall fields with 3 km grid resolution showed a linear relationship with the elevations, which was in good agreement with the relationship introduced by the DLTE, especially when using the latter random cascade generator. The results suggest that the proposed method has the ability to capture statistical characteristics of topography on rainfall fields found in rainfall data.

Acknowledgments: The authors thank the Yodo River Dams Control Office, Ministry of Land, Infrastructure and Transport, Japan for providing radar rainfall data, and Dr. Suzuki at Department of Civil Engineering, Utsunomiya University for making comments about DLTE.

REFERENCES

- [1] Nakakita, E., Y. Suzuki, and S. Ikebuchi (2001) Hierarchical time-scale structure in the dependence of rainfall distribution on topography, *Journal of Hydroscience and Hydraulic Engineering*, Japan Society Civil Engineers, 19 (2), pp. 1-10.
- [2] Suzuki, Y., E. Nakakita, and S. Ikebuchi (2002) A study of dependence properties of rainfall distribution on topographic elevation, *Journal of Hydroscience and Hydraulic Engineering*, Japan Society Civil Engineers, 20 (1), pp. 1-11.
- [3] Waymire, E., V. K. Gupta and I. Rodriguez-Iturbe (1984) A spectral theory of rainfall Intensity at the meso-beta Scale, *Water Resour. Res.*, 20(10), pp. 1453-1465.
- [4] Over, T. M., and V. K. Gupta (1996) A space-time theory of mesoscale rainfall using random cascades, *Jour. Geophys. Res.*, 101(D21), pp. 26,319-26,331.
- [5] Jothityangkoon, C., M. Sivapalan, and N. R. Viney (2000) Tests of a space-time model of daily rainfall in Southwestern Australia based on nonhomogeneous random cascades, *Water Resour. Res.*, 36(1), pp. 267-284.