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Scale Issues in Hydrological Geomorphology and Developing Scale Invariance in Surface Flow Hydrology

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ABSTRACT: It is found that basin hydrological response in relations with the drainage basin dominating geomorphological parameters is directly influenced by the scale of DEM resolution. A Scale Invariant model for the topographic index distribution (Pradhan *et al.*, 2004a) has fulfilled a part of this gap. A scale independent relationship in flood routing models in a distributed hydrological model is yet to be developed. To overcome this problem, scale laws that govern the relation in digital elevation data resolution on upslope contributing area has been analyzed and a mathematical formulation has been derived that successfully downscaled the upslope contributing area from coarse resolution DEM to target fine resolution DEM. The method to downscale the upslope contributing area is used to obtain the similar distribution of depth, cross-section and kinematic wave celerity from different DEM resolutions in Kamishiiba catchment (210 km^2) and to develop a scale invariant model in the surface flow hydrology.

Key Words: *Scale invariant, hydrological geomorphology, topographic index, contributing area, flow routing*

1 INTRODUCTION

Hydrological geomorphology is literally the interface between hydrology, the science of water and geomorphology, the study of landforms and their causative processes. Despite the enormous capacity of today's (and tomorrow's) information technologies, the complexity of the Earth's surface is such that the most voluminous descriptions are still only coarse generalizations of what is actually present. The need for continued and sustained research on scale issues in hydrological geomorphology is therefore self-evident.

Much of the spatial variability can be ignored at "small" spatial scales on the order of 0.1- 1.0m. Indeed, the scientific understanding of individual hydrologic processes at laboratory scales, such as flow through saturated and unsaturated columns of porous media, is fairly well advanced. In particular, one wants to know how the laboratory-scale equations can be spatially integrated so as to describe the hydrologic cycle over a hillside. As the spatial scale under consideration increases to that of a single hillside, spatial variability becomes important, and new elements begin to influence the hydrologic mass balance, such as the topography of the hillside. With the development of the scale invariant model for the topographic index distribution, Pradhan *et al.* (2004a) showed how the laboratory-scale equations can be spatially integrated to provide a consistent hydrologic mass balance in a topography driven model, TOPMODEL.

Beyond a single hillside, a river basin can be viewed as a channel-network-hills system. The hydrologic cycle for larger sub-basins involves the spatially integrated behaviors of several hills along a channel network. An understanding of the spatial variability among hillsides and their interactions through a channel network is necessary for this integration. Thus, at this point scale invariance in surface wave models finds an important component of the hydrologic cycle in river-basin hydrology.

The basic guide line set by this research to obtain physically based hydrological relationships independent of regions and scales is to develop an effective translation method of the scale dependence relations of the dominating hydrological and geomorphologic processes linked to typical properties of the catchment into effective hydrological model. Thus, this research is focused on the development of the scale invariance in catchment hydrology to develop a model consistent with observations. The model can be a potential tool to predict ungauged basins in an effective way.

2 SCALE INVARIANT MODEL FOR TOPOGRAPHIC INDEX (Pradhan *et al.*, 2004a)

It is particularly surface water hydrology that interacts with geomorphology although recently there has been an increasing convergence between research in geomorphology and in groundwater hydrology. In addition to

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relations between drainage basin characteristics and basin hydrological response, geomorphologists have made particular contributions in the investigation of runoff producing areas and the dynamic ways in which such areas contribute to the generation of stream hydrographs, including headwater drainage systems and the modeling of their role in runoff production, TOPMODEL (Beven and Kirkby, 1979).

TOPMODEL allows for spatial heterogeneity by making calculations on the basis of the distribution function of an index of hydrological similarity, soil topographic index, given by Equation (1):

$$SI = \ln \left\{ \frac{a}{To \tan \beta} \right\} \tag{1}$$

where SI is the soil topographic index, a is the area draining through a point, $\tan\beta$ is the local slope angle at that point and To is the local down-slope transmissivity at soil saturation. Points with the same value of the index will be predicted as having the same hydrological responses. Specifying a spatial distribution for To being much more problematic in most applications, it has been assumed to be spatially homogeneous. In the case the similarity index defined by Equation (1) reduces to Equation (2).

$$TI = \ln \left\{ \frac{a}{\tan \beta} \right\} \tag{2}$$

where TI is the topographic index. Topographic index is scale dependent (Zhang and Montgomery, 1994) which leads identified parameter values to be dependent on DEM resolution. When the resolution of DEM change for a catchment, the spatial distribution of topographic index also change. In this scenario keeping the same effective parameter value of To for changed topography index cannot fulfill the hydrological similarity concept of TOPMODEL across different DEM resolutions in the catchment.

Figure 1(a) shows the blunder in simulated result (with Nash efficiency of -45%) of 1000m DEM resolution TOPMODEL, with effective parameter values identified at 50m-grid resolution DEM (refer Table 1). For this purpose notions of scale transformation and scale invariance are needed. We developed a method to downscale topographic index which is given by equation (3):

$$TI_{scaled} = \ln \left(\frac{C_i}{W_i R_f \theta_{scaled}} \right) \tag{3}$$

where TI_{scaled} is the scaled topographic index, C_i is the upslope contributing area of the coarse resolution DEM, W_i is the unit contour length of coarse resolution DEM, i is a location in catchment, R_f is a resolution factor and θ_{scaled} is the downscaled steepest slope of the target resolution DEM by fractal method. Details of the derivation of Equation (3) is given in Pradhan *et al.* (2004a). Equation (3) is coupled with TOPMODEL to develop the Scale Invariant TOPMODEL (Pradhan *et al.,* 2004b). The Scale Invariant TOPMODEL is applied to the Kamishiiba catchment (210 km^2).

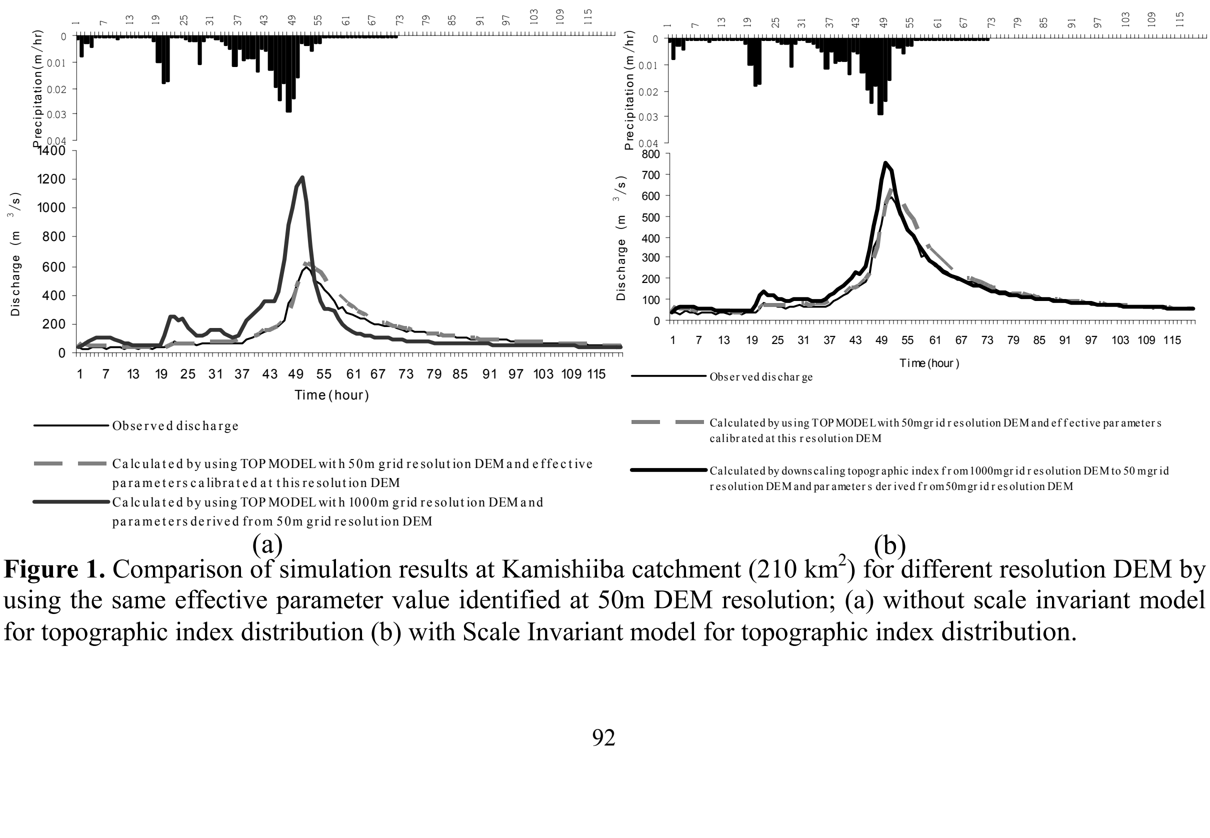


Figure 1. Comparison of simulation results at Kamishiiba catchment (210 km^2) for different resolution DEM by using the same effective parameter value identified at 50m DEM resolution; (a) without scale invariant model for topographic index distribution (b) with Scale Invariant model for topographic index distribution.

Table 1. Effective parameter values identified at 50m DEM resolution in Kamishiiba catchment (210 km^2).

Lateral transmissivity of soil at saturation condition, *To* [m^2/hr]	decay factor of lateral transmissivity with respect to saturation deficit, *m* [m]	Maximum root zone storage, Rzmax [m]	Manning's roughness coefficient *n*
9.8	0.07	0.001	0.037

Figure 1(b) shows that the simulated runoff from Scale Invariant TOPMODEL applied at 1000m-grid resolution DEM, with the same effective parameter values derived from 50m-grid resolution DEM in Table 1, has matched with the simulated runoff of 50m DEM resolution TOPMODEL and also with the observed runoff with Nash efficiency of 90%. The method to downscale topographic index distribution has solved two problems. Firstly, it has given consistency to the physically reliable effective parameter value of *To* with observations at the scale of interest; it can't be denied the fact that physically realistic value of *To* also finds equally important in infiltration excess dominating hydrological processes. Secondly, it has successfully reduced the uncertainty associated with a scale in prediction.

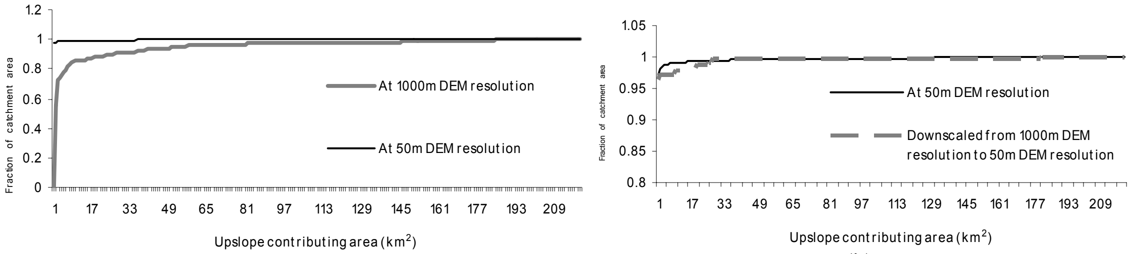
Although the relative importance of the components in the rainfall-runoff transformation process depends both on its working scale and on the geographical, climatic and environmental conditions of the site under consideration, the relative importance of routing phenomenon in surface flow hydrology cannot be ignored for a complete process model that offers a detailed description of the rainfall-runoff transformation. In Figure 1 (b), the same effective parametric value of Manning's roughness coefficient *n* identified at 50m DEM resolution (see Table 1) is used to 1000m DEM resolution without analyzing the scale effects in routing. Thus, the hydrological response of scale invariant TOPMODEL alone in Figure 1(b) still suffers discrepancy from that of 50m DEM resolution hydrological response and the observed discharge by overestimating the peak flows.

3 SCALE EFFECTS IN THE INTERFACE BETWEEN SURFACE FLOW HYDROLOGY AND GEOMORPHOLOGY: ANALYSIS, SOLUTION METHOD, RESULTS AND DISCUSSIONS

Flow routing in channels has been a subject of much discussion for over half a century and more especially since the advent of digital computers. Flow routing is a technique for determining the propagation of flow from one point in the channel to another. Flow routing in open channels entails wave dispersion, wave attenuation or amplification and wave retardation or acceleration. These wave characteristics constitute the hydraulics of flow routing or propagation and are greatly affected by the geometric characteristics of channels. The flow variables whose propagation characteristics are of interest are discharge, velocity, depth, cross-section, volume and duration. In catchment hill slope channel routing these flow variables is a function of upslope contributing area. In the DEM based distributed hydrological model, the smaller contributing area less than a grid area of a coarse DEM resolution used is completely lost. In Figure 2(a), the smaller contributing area less than a km^2 that appears over 97% in 50m DEM resolution is seen completely lost when 1000m DEM resolution is used.

3.1 Development of Scale Invariant model for the upslope contributing area

The density of the small contributing area is higher in a catchment. In Figure 3(a), it is observed that this small



(a) (b)

Figure 2. Comparison of upslope contributing area distribution function from different DEM resolutions in Kamishiiba catchment (210 km^2) (a) without downscaling method for upslope contributing area (b) with downscaling method for upslope contributing area.

contributing area less than a grid area of the coarse resolution DEM used is completely lost. In fact the smallest contributing area derived from a DEM resolution is a single grid of the DEM at that resolution. Thus area smaller than this grid resolution is completely lost as the larger sampling dimensions of the grids act as filter. But as we use finer resolution DEM, the smaller contributing area - that is the area of finer grid resolution is achieved. From this point of view, we introduced number of sub grids N_s to derive scaled upslope contributing area as shown by Equation 4.

$$C_{i\,scaled} = \left(\frac{C_i}{N_s I_f} \right) \tag{4}$$

where $C_{iscaled}$ is the scaled upslope contributing area at a point i, I_f is a influence factor. N_s is the total number of subgrids within a coarse resolution grid. i is a location in a catchment. The area of a coarse resolution grid shown itself is the smallest contributing area for that coarse DEM resolution. When this area of coarse resolution DEM is divided by the number of sub grids (sub grid as target resolution DEM) that together adds up to make the coarse resolution grid, area of a sub grid as smallest contributing area for the target DEM resolution is obtained.
In a catchment as the upslope contributing area gets bigger and bigger, the contributing area values given by coarse and fine resolution DEM at the points downstream becomes closer and closer; thus the influence of N_s on C_i must gradually decrease in Equation (4). For this reason we introduced influence factor I_f in Equation (4) and I_f is described as;

$$I_f = e^{\left\{ \frac{(1-N_i)H}{N_o} \right\}} \tag{5}$$

where, N_i is the number of the coarse resolution grids contained in the contributing area at a location i in the catchment, N_0 is the number of the coarse resolution grids contained in the contributing area at the outlet of the catchment. H in Equation (5) is introduced as harmony factor. Considering the influence of N_s on C_i in Equation (4) is almost negligible at the outlet of the catchment, the value of H can be obtained from Equation (6) as;

$$N_s e^{-H} = 1 \tag{6}$$

Finally, we developed a Scale Invariant model for the upslope contributing area from Equations (4) and (5) as;

$$C_{i\,scaled} = \left(\frac{C_i}{N_s e^{\left\{ \frac{(1-N_i)H}{N_o} \right\}}} \right) \tag{7}$$

Using Equation (7), the upslope contributing area is downscaled from 1000m DEM resolution to 50m DEM resolution. In contrast to Figure 2(a), Figure 2(b) shows the similar distribution of upslope contributing area from 50m DEM resolution and downscaled from 1000m DEM resolution to 50m DEM resolution.

3.2 Development of Scale Invariance in surface flow hydrology

A wave is a variation in flow, such as a change in flow rate or water surface elevation, and the wave celerity is the velocity with which this variation travels along the channel. The kinematic wave celerity, c_k, can be defined in terms of flow depth by Equation (8).

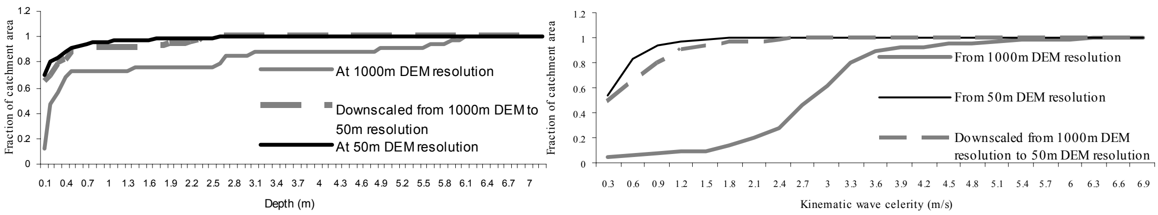
$$c_k = \frac{5}{3}\left(\frac{S_i^{1/2}}{n} \right) y_i^{2/3} \tag{8}$$

where S_i is the slope and n is the Manning's roughness coefficient. y_i is the depth of flow and is expressed as;

$$y_i = \left(\frac{nQ_i}{S_i^{1/2} B_i} \right)^{3/5} \tag{9}$$

where Q_i and B_i are the flow rate and channel width respectively at a point i.

In a distributed system routing the flow is calculated as a function of space and time through the system. Like lateral transmissivity of soil To in Equation (1), the Manning's roughness coefficient n in Equation (8) and (9) is also an effective parameter. Figure 3(a) shows much difference in distribution of depth y_i from 50m DEM resolution and from 1000m DEM resolution when keeping the same value of Manning's roughness coefficient



(a) (b)

Figure 3. Comparison of (a) flow depth distribution function from different DEM resolutions in Kamishiiba catchment (210 km^2) without and with it's downscaling method (b) Kinematic wave celerity distribution from different DEM resolutions in Kamishiiba catchment (210 km^2) without and with it's downscaling method.

for the 1000m DEM resolution, identified at 50m DEM resolution (0.037 in Table 1). This scale problem in depth has serious impact on the distribution of kinematic wave celerity as shown in Figure 3(b).

The root of this problem originates from the scale problem on upslope contributing area as discussed earlier in section 3.1 and in figure 2(a), and also from the underestimation of slope in coarse resolution DEM. Upslope contributing area is a key variable because of its intrinsic capability to describe the nested aggregation structure embedded in the fluvial landforms and its important physical implications (e.g., Rodriguez-Iturbe and Rinaldo, 1997). Flow rate Q_i being a function of upslope contributing area (Strahler, 1964), from Equation (7) we formulate to obtain the flow rate given by target resolution DEM at point i as;

$$Q_{itarget} = \left(\frac{Q_i}{N_s e^{\left\{ \frac{(1-N_i)H}{N_o} \right\}}} \right) \tag{10}$$

The downscaled channel width at a location i, $B_{iscaled}$ is given as;

$$B_{iscaled} = aC_{iscaled}^{\ b} \tag{11}$$

where a and b are the coefficients whose values are independent of the scale of DEM.
Substituting Q_i ,B_i and S_i in Equation (9) by $Q_{itarget}$ and $B_{iscaled}$ from Equations (10) and (11) and by θ_{scaled} (refer Pradhan *et al.*, 2004a) respectively we develop the method to downscale the flow depth, $y_{iscaled}$ as;

$$y_{iscaled} = \left(\frac{nQ_{itarget}}{\theta_{scaled}^{\ 1/2} B_{iscaled}} \right)^{3/5} \tag{12}$$

Finally, substituting y_i and S_i in Equation (8) by $y_{iscaled}$ and θ_{scaled} we developed the method to downscale the wave celerity distribution. In Figure 3 (a) (all the simulation results in Figures 3(a) and 3(b) are made at time step 43 hours of the rainfall event), it is shown that the distribution of downscaled flow depth from 1000 m DEM resolution to 50m DEM resolution and that from 50m DEM resolution (Manning's roughness coefficient n used is identified at 50m DEM resolution) has matched. Thus, by using Equation (12) we successfully reduced the over estimation of depth given by 1000m DEM resolution (also shown in Figure 3(a)). Figure 3(b) shows the perfect match of the distribution of downscaled wave celerity c_k from 1000 m DEM resolution to 50m DEM resolution and that from 50m DEM resolution (n value used is identified at 50m DEM resolution; refer Table 1).

Several variations of the kinematic wave routing method have been proposed. These routing methods can be easily coupled with Equation (11) and Equation (12) to develop a scale invariant routing method. Cunge (1969) proposed that Muskingum method can be considered an approximate solution of a modified diffusion equation. Coupling this routing method with the method to downscale depth and kinematic wave celerity in Equation (11) and (12) we further analyzed the simulation results with the Scale Invariant TOPMODEL as shown in Figure 4. In Figure 4, the simulated result downscaled from 1000m DEM resolution (with downscaling routing method and Scale Invariant TOPMODEL, using the same parameters in Table 1 identified at 50m DEM resolution) has perfectly matched with the simulation result of 50m DEM resolution with further increment in Nash efficiency from 90% (in Figure 1(b)) to 93%.

4 CONCLUSION

There is a long tradition in geomorphology of seeking generalizable rules for landscape evolution such that real landscapes, and particularly their scale-dependent attributes, can be modeled. However, basin hydrological response in relations with the geomorphological parameters are influenced by DEM resolution. We developed

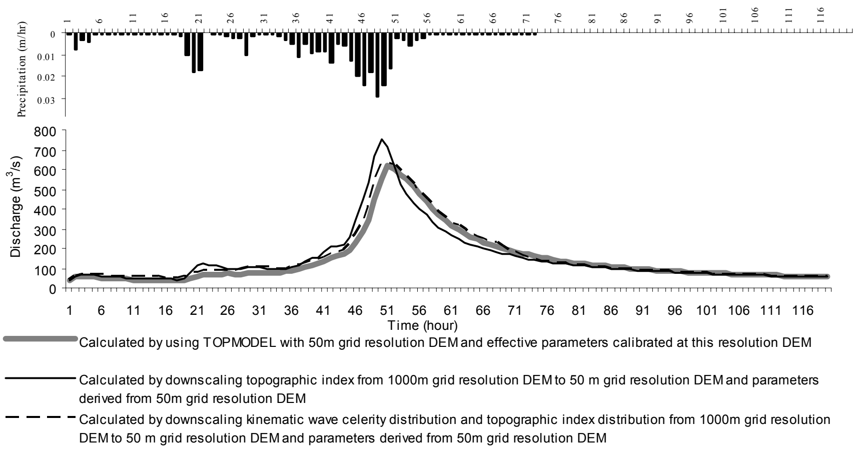


Figure 4. Comparison of simulation results at Kamishiiba catchment (210 km^2) at 50m DEM resolution, at 1000m DEM resolution with downscaled topographic index distribution and at 1000m DEM resolution with downscaled topographic index distribution and kinematic wave celerity. For all the simulations, same parameters identified at 50m DEM resolution is used.

the concept of resolution factor and a fractal method for scaled steepest slope to account for the effect of scale on topographic index distribution. The method to downscale the topographic index is then coupled with TOPMODEL to develop the Scale Invariant TOPMODEL. In this research we analyzed the scale laws that govern the relation in digital elevation data resolution on upslope contributing area and developed a mathematical formulation to downscale the upslope contributing area. The method to downscale the contributing area is successfully applied to downscale the flow variables and developed a scale invariant model in surface flow hydrology. We propose the scale invariant models in basin hydrology as a method to enhance the consistency of a complete process model in rainfall-runoff transformation. This research aims to offer applied opportunities to identify physically based hydrological relationships independent of scale and region (Pradhan *et al*., 2005).

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