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Derivation of Rainfall Intensity-Duration-Frequency Relationships for Short-Duration Rainfall from Daily Data

Le Minh NHAT, Y. Tachikawa, T. Sayama, K. Takara
Disaster Prevention Research Institute, Kyoto University, Uji, Kyoto 611-0011, Japan
nhat@flood.dpri.kyoto-u.ac.jp

ABSTRACT

In this paper, the properties of time scale invariance of rainfall are investigated and applied to Intensity-Duration-Frequency (IDF) relationships. The hypothesis of simple scaling implies in direct and empirically verifiable relations among the moments of several orders of rainfall intensities in different durations. Using these relations, it is possible to analytically derive IDF relationships for short-duration rainfall from the statistical characteristics of daily data only. The simple scaling model has been applied to precipitation data observed at the

Introduction

The intensity-duration-frequency (IDF) relationship of heavy storms is one of the most important hydrologic tools utilized by engineers for designing flood alleviation and drainage structures in urban and urban areas. Local IDF Equations are often estimated on the basis of records of intensities abstracted from rainfall depths of different durations, observed at a given recording rainfall gauging station. In some regions, there may exist a number of recording rainfall gauging stations operating for a time period sufficiently long to yield a reliable estimation of IDF relationships; in many other regions, however, these stations are either non-existent or their sample sizes are too small in developing countries. Because daily precipitation data is the most accessible and abundant source of rainfall information, it seems natural, at least for the regions where data at higher time resolution are scarce, to develop and apply methods to derive the IDF characteristics of short-duration events from daily rainfall statistics. In this regard and in contrast to earlier empirical disaggregation techniques, the works of Burlando & Rosso (1996), Menabde et al. (1999) and Pao-Shan Yu et al.(2004) are examples of methodologies in which the theories of scaling properties and employed to infer the IDF characteristics of short-duration rainfall from daily data.

This paper aims

1. To present the properties of time scale invariance of rainfall,
2. To apply them to a location of station in Yodo cathment of Japan,
3. To derive a IDF relationship for short-duration rainfall from daily data, and
4. To compare with traditional method and discuss the results.

The paper is organized as follows: The next two sections present the theoretical background as related to the time scale invariance properties of short-duration rainfall. In the sequence, these properties are verified for data observed at rainfall gauging stations located in the Yodo catchments, in Japan. The IDF relationship, derived from daily rainfall data, is then compared to the traditional estimated curve. Conclusions are given in the last section.

The generalized IDF relationship

In recent years, the study of phenomena with scale variance has grown from applications in physics phenomena such as statistical theories of turbulence field theory (Gupta and Waymire, 1990) to hydrologic phenomena such as stochastic rainfall modeling and intensity-duration-frequency (IDF) curve formulation. While empirical equations have been used for nearly one hundred years to explain the form of IDF curves, scale invariance has helped to understand these relationships. Sherman (1905) first developed a generalized IDF relationship, and many other versions of this relationship have been developed in the years since. All forms of the generalized IDF relationships assume that rainfall depth or intensity is inversely related to the duration of a storm raised to a power, or scale factor.

The IDF formulas are the empirical equations representing a relationship among maximum rainfall intensity (as dependant variable) and other parameters of interest such as rainfall duration and frequency (as independent variables). There are several commonly used functions found in the literature of hydrology applications (Chow et al., 1988). Four basic forms of equations used to describe the rainfall intensity duration relationship are summarized:

Talbot Equation:

$$i = \frac{a}{d + b} \quad (1)$$

Bernard Equation:

$$i = \frac{a}{d^e} \quad (2)$$

Kimijima Equation:

$$i = \frac{a}{d^e + b} \quad (3)$$

Sherman Equation:

$$i = \frac{a}{(d + b)^e} \quad (4)$$

where *i* is the rainfall intensity (mm/hour); *d* is the duration (minutes); *a, b* and *e* are the constant parameters related to the metrological conditions.

Although many previous studies depend on curve-fitting techniques, studies generalizing IDF rainfall formulas have become popular over the past 20 years. These studies include Hershfield (1961) developed various rainfall contour maps to provide the design rain depths for various return periods and durations. Bell (1969) proposed a generalized IDF formula using the one hour, 10 years rainfall depths; P_1^{10}, as an index. Chen (1983) further developed a generalized IDF formula for any location in the United States using three base rainfall depths: P_1^{10}, P_{24}^{10}, P_1^{100}, which describe the geographical variation of rainfall. Kouthyari and Garde (1992) presented a relationship between rainfall intensity and P_{24}^2 for India.

Koutsoyiannis et al. (1998) have updated the IDF relationship, for given return period, IDF relationships are particular cases of the following general empirical formula:

$$i = \frac{w}{\left(d^{v} + \theta\right)^{\eta}} \quad (5)$$

where *i* denotes the rainfall intensity for duration *d* and *w, v, θ,* and *η* represent non-negative coefficients. A numerical exercise proposed by Koutsoyannis et al. (1998) shows that the errors resulting from imposing v=1 in Equation (5) are much smaller than the typical parameter and quantile estimation errors from limited size samples of rainfall data. Considering the specification of $v \neq 1$ results in over-parametrization of Equation (5), Koutsoyannis et al. (1998) suggested the following equation as a general expression of IDF relationships for a given return period:

$$i = \frac{w}{(d + \theta)^{\eta}} \quad (6)$$

Rigorously, the coefficients *w, θ* and *η* in Equation (6) are not independent from the return period. However, because the IDF curves for different return periods cannot intercept each other, such a dependence cannot be arbitrary; this restriction imposes limits to the variation range of parameters *w, θ* and *η*. For instance, if *{w1, θ1, η1}* and *{w2, θ2, η2}* denote two different parameter sets for return periods T1 and T2 < T1 respectively, then Koutsoyannis et al. (1998) suggest the following possible restrictions to the parameter space:

$$\theta_1 = \theta_2 = \theta \geq 0; 0 < \eta_1 = \eta_2 = \eta < 1; w_1 > w_2 > 0 \quad (7)$$

In this set of restrictions, note that the only parameter that can consistently increase with increasing return periods is w, which results in substantial simplification of Equation (6). In fact, these arguments justify the formulation of the following general model for IDF relationships:

$$i = \frac{a(T)}{b(d)} \quad (8)$$

which possesses the great advantage of presenting separable relationships between i and T and between i and d. In Equation (8), $b(d) = (d + \theta)^{\eta}$ with $\theta > 0$ and $0 < \eta < 1$, whereas *a(T)* is completely defined by the probability distribution function of the maximum rainfall intensity. The form of Equation (8) is consistent with most IDF empirical equations estimated for many locations: For example L.M.Nhat et al. (2006) established the IDF curves for precipitation in the monsoon area of Vietnam.

The scaling of rainfall intensity theory

In this section, a general theoretical framework for the proposed model is introduced. Let the random variable I(d) the maximum annual value of local rainfall intensity over a duration d. It is defined as:

$$I(d) = \max_{0 \le r \le 1\,year} \left[\frac{1}{d} \int_{1-d/2}^{1+d/2} x(\xi) d\xi \right] \quad (9)$$

where X *(ξ)* is a time continuous stochastic process representing rainfall intensity and d is point in time. Suppose that I*(d)* represents the Annual Maximum Rainfall Intensity (AMRI) of duration *d*, defined by the maximum value of moving average of width *d* of the continuous rainfall process. The random variable I*(d)* has a cumulative probability distribution, which is given by

$$\Pr(I_d \le i) = F_d(i) = 1 - \frac{1}{T(i)} \quad (10)$$

Here, some concepts are introduced about scaling of the probability distribution of random functions (Burlando & Rosso, 1996, Menable, 1999). A generic random function I(*d*) is denoted by simple scaling properties if obeys the following

$$I_d = \left(\frac{d}{D} \right)^{-H} I_D \quad (11)$$

Defining $\lambda = \frac{D}{d}$ as the scaling ratio

$$I(d)^{dist} = \lambda^H I(\lambda d) \quad (12)$$

where H is a non-integer scaling exponent factor.

The equality " $\overset{dist}{=}$ " refers to identical probability distributions in both side of the equations.

The Equation (12) may be rewritten in terms of the moments of order q about the origin, denoted by E[I_d^q]; in these terms, the resulting expression is

$$E\left[I_d^q\right] = \lambda^{Hq} E\left[I_{\lambda d}^q\right] \quad (13)$$

The scaling exponent, H*q*, can estimated from the slope of linear regression relationship between the log-transformed values of moment $\left(\log\ E\left[I_{\lambda d}^q\right]\right)$ and scale parameters (log λ) for various order of moment *(q)*. This is definition of "strict sense" simple scaling (*Gupta & Waymine* 1990). A less restrictive definition is "wide sense" simple scaling with $d = 1$ hour and $\lambda d = \lambda$, given by

$$E\left[I_1^q\right] = \lambda^{Kq} E\left[I_\lambda^q\right] \quad (14)$$

The scaling exponent H is not constant with the order of moment *(q),* as Equation (14), but it varies as in:

$$E\left[I_1^q\right] = \lambda^{K(q)} E\left[I_\lambda^q\right] \quad (15)$$

The *K(q)* is function of the moment order. The procedure is adopted to test the suitability of scale invariant model to describe rainfall process, in here briefly described in Figure 1. The same moment $E\left[I_{\lambda d}^q\right]$ are plotted on the logarithmic chart versus the scale λ for different moments' order q_i. The slope $K(q_i)$ is plotted on the linear chart versus the moment order q_i. If the resulting graph is a straight line, the field is simple scaling, while in opposite case of curved line, multi-scaling is observed.

According to Menabde et al. (1999), the only function form of $E\left[I_{\lambda d}^q\right]$ is capable of satisfying equation:

$$d^{Hq} E\left[I_d^q\right] = (\lambda d)^{Hq} E\left[I_{\lambda d}^q\right] \quad (16)$$

$$F_d(i) = F_{\lambda d}\left[\lambda^H i\right] \quad (17)$$

$$E\left[I_d^q\right] = K(q) d^{Hq} \quad (18)$$

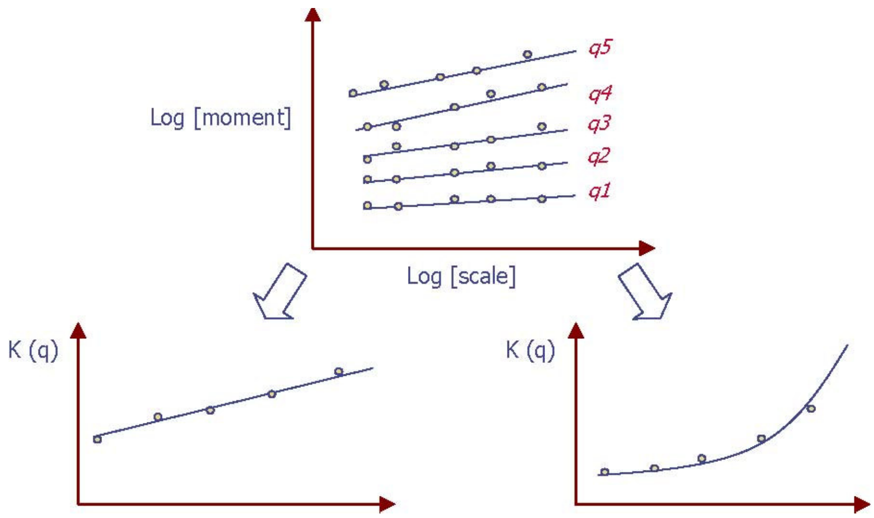
For many parametric forms, Equation (18) may be expressed in terms of standard variant, as in

$$F_d(i) = F\left[\frac{i - \mu_d}{\sigma_d}\right] \quad (19)$$

Where F(.) is a function independent of *d.* Under this form, it can be deduced from Equation (18) that

$$\mu_d = \lambda^H \mu_{\lambda d} \quad (20)$$

$$\sigma_d = \lambda^H \sigma_{\lambda d} \quad (21)$$



Simple scaling Multi-scaling

Figure. 1 Simple and multiscaling in term of statistical moments. Fist step, moments of different orders q are plotted as function of scale in a log-log plot. From the slope, values of the function K(q) are obtained. If K(q) is linear, the process is simple scaling. If K(q) is non-linear, the process is multiscaling.

Substituting Equation (19), (20) and (21) into Equation (8) and investing with respect to i, one obtains:

$$i_{d,T} = \frac{\mu_{\lambda d}(\lambda d)^H + \sigma_{\lambda d}(\lambda d)^H F^{-1}(1-1/T)}{d^H} \quad (22)$$

By equaling Equation (22) to the general model for IDF relationship, given by Equation (7), it is easy to verify that

$$H = \eta \quad (23)$$

$$\theta = 0 \quad (24)$$

$$b(d) = d^{\eta} \quad (25)$$

$$d(T) = \mu_{\lambda d}(\lambda d)^H + \sigma_{\lambda d}(\lambda d)^H F^{-1}(1-1/T) \quad (26)$$

$$i_{d,T} = \frac{\mu + \sigma F^{-1}(1-1/T)}{d^{\eta}} \quad (27)$$

where: $\mu = \mu_{\lambda d}\ (\lambda d)^H$ and $\sigma = \sigma_{\lambda d}\ (\lambda d)^H$ are constants. It is worthwhile to note that the simple scaling hypothesis leads to the equality between the scale factor and the exponent in the expression relating rainfall intensity and duration.

The simple scaling property, as formalized by Equation (17), can be empirically verified by replacing the population moments by the corresponding sample moments. On the other hand, in order to verify the validity of Equation (18), one needs to specify a probability distribution for the annual maximum rainfall intensities. In such a context, two of the most frequently used probability distributions, namely the Generalized Extreme Value (GEV) and the EV1 or Gumbel parametric functions, are examples of functional forms that are compatible with expression (19) and appropriate for the empirical verification of Equation (18).

Application simple scaling in time series of rainfall This section describes the application of the simple scaling model for rainfall intensities, as prescribed by Equation (27), to rainfall data observed at the Hikone and Hirakata rainfall station of Yodo catchments, located in Japan.

A first attempt to verify the simple scaling hypothesis has been made by using sample moments in Equation (17), with $H = \eta$. Figure 2 depicts the resulting relationship between sample moment order q and duration d, both in logarithmic coordinates, for sample moments of order $q = 1, 2$, up to 5. In this Figure 2, it is clear that for all moment orders considered, there are well defined scale relationships for durations comprised between 1 hour and 24 hours.

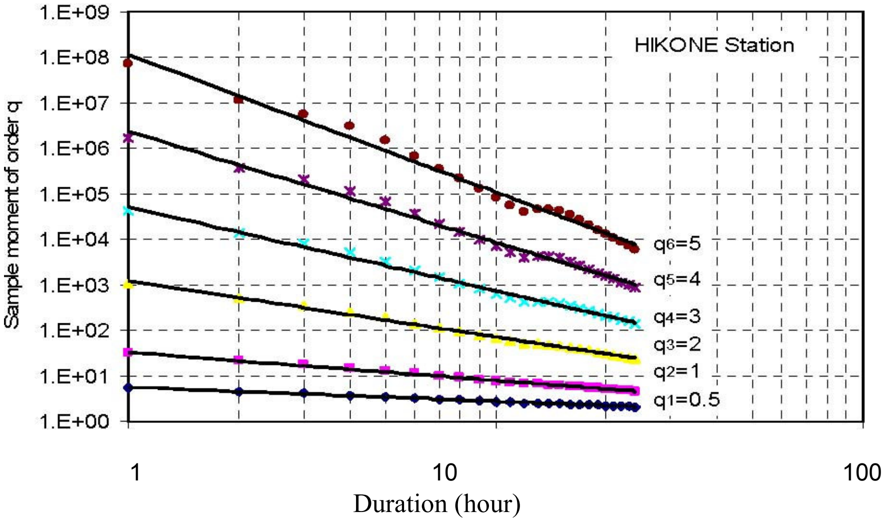


Figure 2 Relationship between sample moments of order q and duration

The regression between moment $E[I_{\lambda d}]$ and duration d, for 1h to 24 h, may also be used to check the validity of Equation (17). In fact, by representing the exponent by $K(q) = -\eta q$ and plotting it against q, Figure 3 shows there is an almost perfect linearity between the two variables. Such a linear relationship confirms the hypothesis of " simple scaling in wide sense", as defined by Equation (17).

The slope of the regression line between $K(q)$ and q is η=0.6058 for Hikone and η=0.678 for Hirakata station, as an estimated for the scale factor.

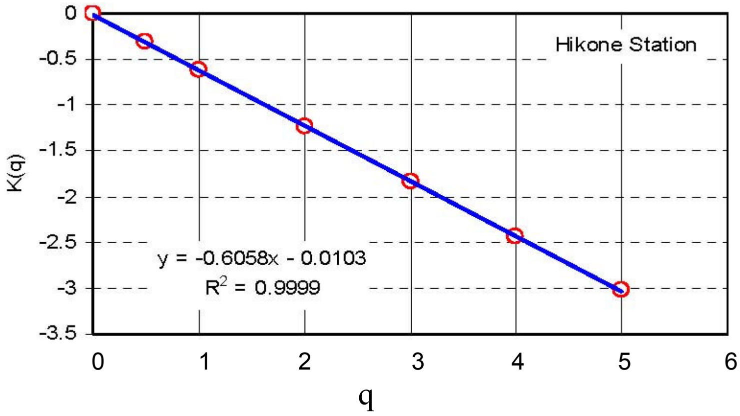


Figure 3 Relationship between K(q) and the sample moment order q

The IDF relationship for short duration rainfall can be deduced form daily data by applying Equation (22) with η=0.6058 for the Hikone station and with the estimates of μ_D and σ_D with D=24h. From 24-hour data collected at the Hilkone recording gauging station, the sample of 21 years of 24hours annual maximum rainfall intensity yields. the estimates $\mu_{D=24}$ = 4.615 and $\sigma_{D=24}$ = 2.604 . Back with these estimates and the Gumbel inverse function, the deduced IDF relationship for the location of Hikone may be written as Equation (30) with $\mu=\lambda^{\eta}\mu_{24}$= 31.56 and $\sigma=\lambda^{\eta}\sigma_{24}$=17.81

$$i_{d,T} = \frac{\mu_{\lambda d}(\lambda d)^{\eta} + \sigma_{\lambda d}(\lambda d)^{\eta} F^{-1}(1-1/T)}{d^{H}} \quad (28)$$

and:

$$i_{d,T} = \frac{\mu + \sigma F^{-1}(1-1/T)}{d^{\eta}} \quad (29)$$

$$i = \frac{31.56 - 17.8 In(- In(1-1/T))}{d^{0.605}} \quad (30)$$

Another traditional way of constructed rainfall IDF curves by Equation (2) (Bernard Equation). Frequency analysis techniques are used to develop the relationship between the rainfall intensity, storm duration, and return periods from rainfall data. Analysis of distribution for rainfall frequency is based on the EVI (Gumbel) distribution. The probability distribution function for EVI is

$$f(x) = \frac{1}{\beta}\left[\frac{x-\alpha}{\beta} - \exp\left(\frac{x-\alpha}{\beta}\right)\right] \qquad (31)$$

The cumulative density function (CDF)

$$F(x) = 1 - \exp\left(\frac{x-\alpha}{\beta}\right) \qquad (32)$$

Where αand βare the parameters. The EVI distribution used to calculate the rainfall intensity at different rainfall durations and return periods and the maximum rainfall intensity for considered durations and 2, 5, 10, 20, 50,100 and 200 years return periods, have been determined. The set of IDF curves can be estimated by Bernard Equation shown in Figure 4.

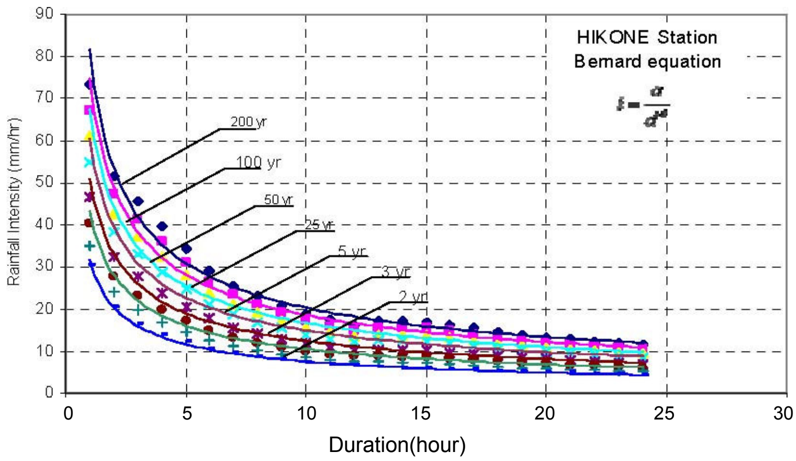
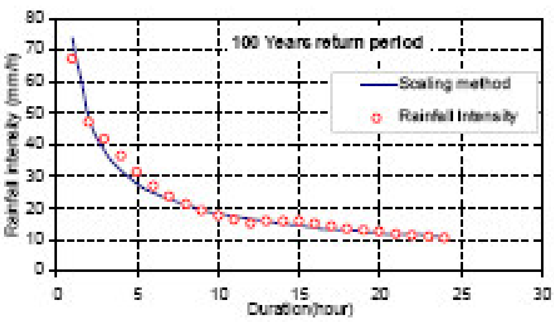
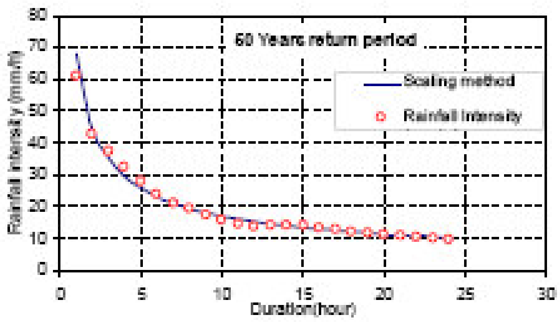


Figure 4 The Rainfall Intensity-Duration-Frequency (IDF) curves for Hikone station by Bernard Equation.

The Rainfall Intensity Duration Frequency curves for Hikone station can be reconstructed be scaling methods by Equation (30), it is show small differences, with higher values for increasing return periods. Figure 5 shows how the IDF relationships, regional climatic characterize.

as calculated by Equation (30) and plots as the return period varies from 5 to 100 year returns. The scale factor η, along with parameters σ and μ in Equation (29), may be interpreted as





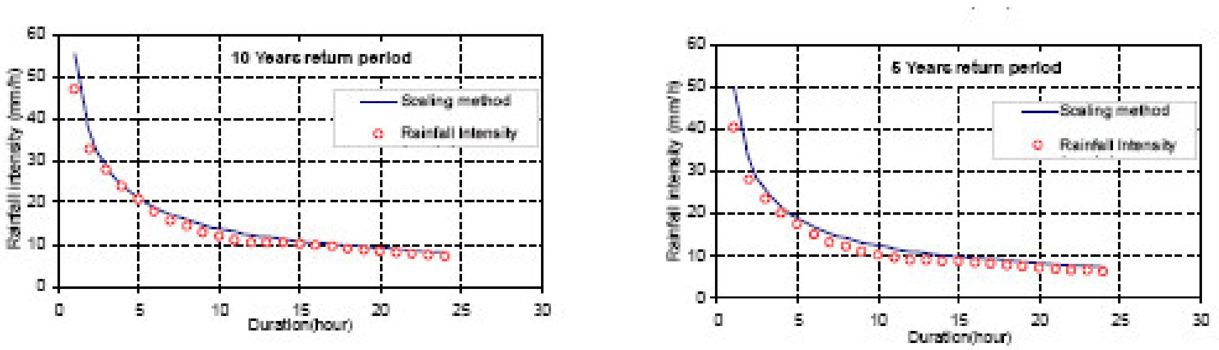


Figure 5 The Rainfall Intensity Frequency Curves at Hikone station, Yodo Catchments, Japan by scaling method.

Conclusions

In the paper, a simple analytical formulation for rainfall IDF relationship, which utilizes the scaling behavior is presented. The proposed approach is based on scaling properties of rainfall time series. The hourly IDF curves were derived in a normalized form apart from EVI (Gumbel) distribution fitted to the maximum rainfall intensity for several durations between 1 hour and 24 hours collected in 2 stations at Yodo catchments of Japan.

The IDF curves for short duration (hourly) were derived from 24-hour data. The simple scaling property verified by local data; then IDF relationships are deduced from daily rainfall which show good results as compared to IDF curves obtained from at-site short-duration rainfall data.

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