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KINEMATIC WAVE FLOW MODELS FOR RIVER BASIN RUNOFF SIMULATION

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Kinematic wave flow models for simulating river basin runoff are reviewd. After the baic equations of kinematic wave flow are derived, some modified kimematic wave flows which are devised for treating surface-subsurface flow on watersheds are introduced. Then, the routing of kinematic wave flows on digital terrain models are inroduced. Finally, lumping methods of kinemtatic wave flows for watershed runoff are shown.

Key Words : *Kinematic Wave Flow, Surface-Subsurface Flow, Lumping*

1. Introduction

Rainfall-runoff processes are water flow phenomena that occur on and beneath the ground surface, and can be modeled using differential equations to describe movement of flowing water. The kinematic wave flow model is one of rainfall-runoff models which have been constructed based on such a notion.

By applying the kinematic wave flow model for surface flow on mountaneous slopes, Ishihara and Takasao[1)] developed the theory of surface runoff and initiated modern physically-based hydrology. In their theory, the discharge of surface flow is assumed to be expressed by the power-law funcion of the depth of the flow. However, as was shown later by themselves[2)], there exists saturated overland flow which appears when the subsurface flow which flows through higly permeable soil layers on montain slopes comes over the layer. To express such flow, it is convenient to express both the saturated overland flow and the subsurface flow in an integrated fashion. Thus in Section 2, we introduce more general form of kinematic wave flow model and its solution procedure. In Section 3, the applicaions of those general form of kinematic flow model to modeling watershed runoffs are introduced. Section 4 explains the routing of the kinematic wave flow on digital terrain models of river basins. Finally, Section 5 shows a lumping method of kinematic wave flows for watershed runoff.

2. BASIC EQUATIONS OF THE KINEMATIC WAVE FLOW

The kinematic wave model is an unsteady open channel flow wave model which is often used in engineering hydrology. The kinematic wave model was originally developed for river routing, but later was applied to catchment rainfall-runoff modeling and several variations of the kinematic wave model were developed.

The basic form of the kinematic wave equation is

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_L(x,t) \tag{1}$$

$$Q(x,t) = \alpha A(x,t)^m \tag{2}$$

where $A(x,t)$ is the flow cross-sectional area, $Q(x,t)$ is the flow discharge, $q_L(x,t)$ is the lateral inflow discharge per unit length, and α and m are constants. $A(x,t)$ and $Q(x,t)$ are abbreviated to A and Q, respectively in Eq.(1).

Equation (2) is derived from Manning's or Chezy's laws, which are flow resistance laws of open channel uniform flow. Manning's law is described as:

$$v = \frac{1}{n} R^{2/3} \sin\theta^{1/2} \tag{3}$$

in which v is the average flow velocity, n is Manning's friction coefficient, R is the hydraulic radius, and θ is the bed slope. By assuming that R

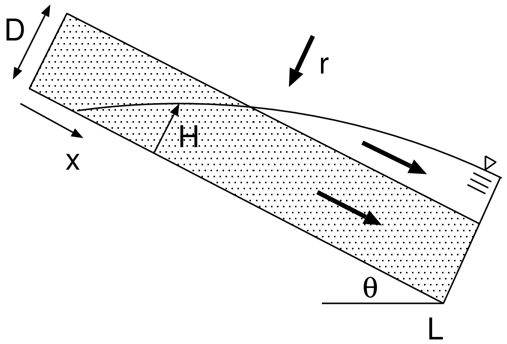

Fig. 1 Schematic representation of surface-subsurface flow on a hillslope

can be generally expressed as $z_1 A^{z_2}$ (z_1 and z_2 are constants), we obtain the expression of Eq.(2).

By applying the kinematic wave approximation to the sheet flow on a plane, we can get also the kinematic wave flow model of surface flow on mountain slopes.

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(x,t) \quad (4)$$

$$q = \alpha h^m \quad (5)$$

in which h is the water depth of the surface flow, q is the discharge per unit width of the slope, r is the lateral input.

It is possible to use the more general form of discharge and depth relation

$$q = f(h,x) \quad (6)$$

as we show in Section 3.

3. SURFACE-SUBSURFACE FLOW MODEL AND CONSIDERATION OF FIELD CAPACITY

Takasao *et al.*[4),5)] developed a kinematic wave model which considers both surface and subsurface flow, and its interaction. This model can describe a saturation excess overland flow mechanism.

They assumed that hillslope surface was covered by a soil layer with high surface infiltration capacity and permeability, which is illustrated in **Fig. 1**. In this figure, D is the depth of the soil layer, L is the length of the hillslope, H is the depth of water flow, and r is the rainfall intensity. Let γ be the porosity of the soil layer and be spatially constant. Then the depth of the effective pore of the soil layer, d, becomes γD. Also, the actual water depth (water volume in height), h, becomes γH $(0 \leq H < D)$ or $H - D + \gamma D$ $(H \geq D)$. Takasao *et al.* derived the following piecewise relation between h and q from Darcy's law and Manning's law:

$$q = \begin{cases} ah & (0 \leq h < d) \\ \alpha(h-d)^m + ah & (h \geq d) \end{cases} \quad (7)$$

where a $(= k\sin\theta/\gamma)$ is the velocity of water which flows in the soil layer, k is the hydraulic conductivity, and α and m are $\sqrt{\sin\theta}/n$ and $5/3$, respectively. The upper relation given in Eq.(7) is used for the case where only subsurface flow occurs, and the lower relation in Eq.(7) is used for the case where saturation excess overland flow occurs.

Shiiba *et al.*[6)] developed a new kinematic wave model by which both flood and low flow simulation can be conducted by introducing a concept of field capacity of hillslope soil layer to the kinematic wave model for surface-subsurface flows. This new kinematic wave model enables a runoff simulation to be connected continuously from flood period to low flow period, or conversely from low flow period to flood period. This model assumes that a part of water which infiltrates into a surface soil layer is mainly captured by the capillary force of soil particles and lateral water flow occurs when water content exceeds the field capacity of the soil.

The kinematic wave equation considering field capacity is as follows:

$$q = \begin{cases} ah_f & (0 \leq h_f < d) \\ \alpha(h_f - d)^m + ah_f & (h_f \geq d) \end{cases} \quad (8)$$

$$h = \begin{cases} h_f + h_c \left(1 - \left(\dfrac{d - h_f}{d}\right)^N\right)^{1/N} \\ \qquad (0 \leq h_f < d) \\ h_f + h_c \quad (h_f \geq d) \end{cases} \quad (9)$$

where q is the discharge per unit width, h is the total water content, h_f is the free water content, a $(= k\sin\theta/\gamma_e)$ is the water flow velocity in a surface soil layer, k is the hydraulic conductivity, γ_e is the effective porosity of a surface soil layer, d $(= \gamma_e D)$ is the depth of pore contained in a surface soil layer, D is the depth of a surface soil layer, $\alpha = \sqrt{\sin\theta}/n$, θ is the slope gradient, n is the Manning roughness coefficient, m is a constant (>1), N is a constant (>1), $h_c = \gamma_c D$, and γ_c is the field capacity of a surface soil layer.

4. ROUTING OF KINEMATIC WAVE FLOW ON DIGITAL TERRAIN MODELS

In recent years, digital elevation maps have become available at a resolution fine enough to clearly describe the topography of catchments.

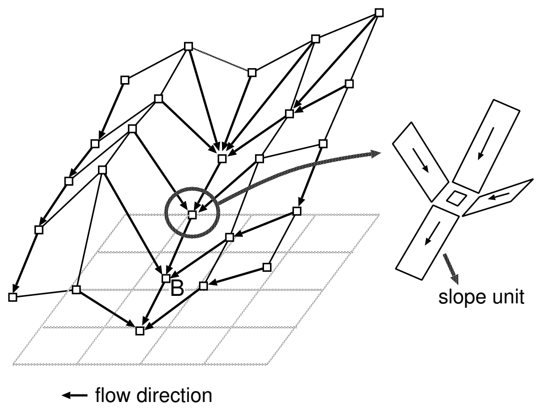

Fig. 2 Schematic representation of the grid-based DEM

Using these data, a number of digital elevation models (DEMs) have been developed to precisely and efficiently represent catchment topography.

Tachikawa *et al.*[7] developed a distributed hydrological model based on a triangular irregular network DEM.

Ichikawa *et al.*[8] applied the kinematic wave model considering field capacity to a grid-based DEM[9]. **Fig. 2** shows a schematic representation of the grid-based DEM which they used. In this figure two adjacent grid points are connected by a rectangular plane. They call this rectangular plane "a slope unit". Catchment topography is represented by a set of slope units. For each slope unit, its area, length and gradient can be easily calculated. They applied the kinematic wave model considering field capacity to all the slope units comprising the catchment topographic model and routed water flows from upstream slope units to downstream slope units. The outflow discharge from the catchment was used as the lateral inflow condition of a river routing model. From the result of the simulaion, Ichikawa *et al.* showed the fact that the kinematic wave model considering field capacity has high applicability to low flow periods.

5. LUMPING KINEMATIC WAVE FLOW EQUATIONS

The distributed runoff models, such as the kinematic wave models, have the advantage that they are based on the physical processes of flowing water. However, generally speaking, they require much computation time and resources. In this section, lumped kinematic wave runoff models are introduced. They are derived by spatially integrating the kinematic wave models. The resulting lumped kinematic wave runoff models reduce the computational burden of water flow routing. These lumped models have a physical basis and the parameter values are calculated using topographic variables obtained from the DEM.

Owing to the limit of space, only the procedure of the lumping of the power-law kinematic wave model is introduced. In this section, the kinematic wave model, $A = KQ^P$ (A: flow cross-sectional area, Q: flow discharge, K and P: constants), is used to describe shallow water flow on a rectangular plane instead of Eq.(5).

The main purpose of this section is to derive a steady-state relation between storage volume and outflow discharge of a rectangular plane by spatially integrating the kinematic wave model. When a rainfall-runoff system reaches a steady condition, the discharge at some point, Q, is expressed as $r \times M$, in which r is the rainfall intensity and M is the upslope contributing area at the point.

Suppose that a catchment is represented by N slope units. Let us consider the ith slope unit ($i = 1, 2, ..., N$). Let L_i be the length, B_i the width, M_i the area, θ_i the gradient, and K_i and P_i the kinematic constants K and P, of the ith slope unit. Also let U_i be the upslope contributing area at the upper end of the ith slope unit. If the ith slope unit is located on the upper boundary of a catchment, $U_i = 0$.

Let $A_i(x)$ be the flow cross-sectional area and $Q_i(x)$ the flow discharge at the distance x from the upper end of the ith slope unit. Since a slope unit is a rectangular plane, the upslope contributing area at the distance x from its upper end of the ith slope unit, $M_i(x)$, is expressed as:

$$M_i(x) = \frac{M_i}{L_i} x + U_i \qquad (10)$$

Then we can obtain

$$Q_i(x) = r \left(\frac{M_i}{L_i} x + U_i \right) \qquad (11)$$

and

$$A_i(x) = K_i r^{P_i} \left(\frac{M_i}{L_i} x + U_i \right)^{P_i} \qquad (12)$$

Spatially integrating $A_i(x)$, we can obtain the storage volume of the ith slope unit, s_i

$$\begin{aligned} s_i &= \int_0^{L_i} A_i(x) dx \\ &= \frac{r^{P_i}}{P_i + 1} \frac{K_i L_i}{M_i} \left\{ (M_i + U_i)^{P_i+1} - (U_i)^{P_i+1} \right\} \end{aligned} \qquad (13)$$

Summing up s_i, we can obtain the total storage volume of the catchment, $S = \sum_{i=1}^{N} s_i$ as

$$S = \sum_{i=1}^{N} \frac{K_i L_i}{(P_i+1)M_i} \left\{ (M_i + U_i)^{P_i+1} - (U_i)^{P_i+1} \right\} r^{P_i} \tag{14}$$

If we assume that the constant in the kinematic wave model, P_i, is spatially uniform and rewrite P_i as P, Eq.(14) can be further simplified as:

$$S = Er^P \tag{15}$$

where

$$E = \frac{1}{P+1} \sum_{i=1}^{N} \frac{K_i L_i}{M_i} \left\{ (M_i + U_i)^{P+1} - (U_i)^{P+1} \right\} \tag{16}$$

Let O be the outflow discharge from the catchment. Under the steady-state condition, $O = r \times \sum_{i=1}^{N} M_i$, rearranged for,

$$r = \frac{O}{\sum_{i=1}^{N} M_i} \tag{17}$$

Substituting Eq.(17) into Eq.(15), we can obtain

$$S = E \left(\frac{O}{\sum_{i=1}^{N} M_i} \right)^P = FO^P \tag{18}$$

where $F = E/(\sum_{i=1}^{N} M_i)^P$.

Equation (18) gives the steady-state relation between the storage volume and outflow discharge of a catchment. However we use this relation to calculate outflow discharge for the unsteady condition, that is, time variant rainfall case. Thus we have the relationship between storage and outflow as follows:

$$S(t) = FO(t)^P \tag{19}$$

The continuity equation of storage volume is given by

$$\frac{dS}{dt} = Q_L(t) - O(t) = Q_L(t) - \left(\frac{S(t)}{F} \right)^{1/P} \tag{20}$$

where $Q_L(t) = r(t) \times \sum_{i=1}^{N} M_i$. Since Eq.(20) is an ordinary differential equation for $S(t)$, we numerically solve this equation by using Runge-Kutta method. We call the model derived here as "the lumped kinematic wave model". The value of F in Eq.(18) can be calculated from the topographic quantities and kinematic constants. This means that the lumped kinematic wave model is a lumped model but the parameters can be specified on a physical basis.

The procedure of lumping the surface-subsurface model is shown in Ichikawa *et al.*[10], and the procedure of lumping the kinematic wave model considering field capacity is shown in Ichikawa *et al.*[11].

6. CONCLUDING REMARKS

Mainly the researches done by the authors about the kinematic wave flow were introduced.

Ichikawa[12], written in English, covers the topics introduced in this paper.

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