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ESTIMATION OF INTENSITY-DURATION-AREA-FREQUENCY CURVES USING SCALING PROPERTIES OF HOURLY RAINFALL DATA

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ABSTRACT

Many hydrological and meteorological applications require knowledge about spatial and temporal variabilities of rainfall over an area. The intensity of point precipitation is only applicable for relatively small areas. For larger areas, design rainfall needs to be converted to average areal depths. Areal Reduction Factors (ARFs) have been commonly used to obtain this transformation. To estimate ARFs at sparely gauged basins, to derive the Intensity-Duration-Area-Frequency (IDAF), it is essential to incorporate the scaling properties of rainfall in time and space.

The IDAF curves are determined for the evaluation of design rainfall using a scaling approach. The variabilities of annual maximum rainfall intensity in area and duration are represented through the scaling properties in time and space. Thus the scaling relationships of mean rainfall intensity with area and duration are derived using the concepts of scaling properties. Using Generalized Extreme Value (GEV) the authors obtain a scaling model to give the IDAF curves of extreme rainfall. An application is made to the Yodo River catchment of Japan.

Keywords: extremes rainfall, scale-invariance, IDF curves, ARF, design rainfall

1. INTRODUCTION

Many hydrological and meteorological applications require knowledge about spatial and temporal variabilities of rainfall over an area. The intensity of point precipitation is only applicable for relatively small areas. For larger areas, design rainfall needs to be converted to average areal depths. Areal Reduction Factors (ARFs) have been commonly used to obtain this transformation. To estimate ARFs at sparely gauged basins, to derive the Intensity-Duration-Area-Frequency (IDAF), it is essential to incorporate the scaling properties of rainfall in time and space.

In the literature, two different types of ARFs are found (Omolayo, 1993; Sivapalan and Blöschl, 1998): The storm-centered ARFs and the fixed-area ARFs. The storm-centered ARFs are associated with rainfall intensity within the rainfall isohyets of specific storm events, they represent the ratio of average storm depths over an area (defined by rainfall isohyets) and the maximum rainfall depths for the storm (at storm centred). The storm-centered ARFs are used more commonly in PMP (probable maximum flood) estimation. The fixed-area ARFs relate rainfall estimation at point to the average over catchment which is fixed in space. They are estimated by constructing from all available rainfall data at station, the time series of catchment average rainfall, performing the same types of extreme value analyses described above for constructing point IDF curves, and finally relating the catchment rainfall intensities to the point values, for the same return period and duration. This study is concerned with estimating fixed area ARFs.

In hydrological risk analysis and design, one is often interested in the rainfall intensity averaged over a region of area A and duration D, with return period T. Plotting such extreme rainfall intensity I(A, D, T) against D for given A and T produces so-called Intensity Duration Area Frequency (IDAF) curves. For $A \rightarrow 0$ (precipitation at a point), the IDAF curves reduce to the familiar Intensity Duration Frequency (IDF) curves.

For a given location, the ARFs can be defined as ratio between the mean rainfall intensity I(T,A) and that of point rainfall intensity.

$$ARF(D, A) = \frac{I(D, A)}{I(D, 0)} = \frac{IDAF}{IDF} \tag{1}$$

Some studies have derived the properties of the IDAF curves and the ARFs using non-scaling representations of rainfall. An early attempt in this direction was made by Rodriguez-Iturbe and Mejia, 1974 approach by assuming that the rainfall field is a zero mean stationary Gaussian process. A different approach to ARF estimation, based on crossing properties of random fields, was proposed by Bacchi and Ranzi (1996). Properties of extremes of random functions were used also by Sivapalan and Blöschl (1998). Finally, Asquith and Famiglietti (2000) derived the ARF as the catchment average of the ratios between the T-year rainfall depths at distance r from the centroid of the storm and at the centroid itself.

In recent years, the concepts of rainfall scale-invariance have come to the fore in both modeling and data analysis in hydrological precipitation research (Gupta and Waymire, 1990; Nguyen *et al.*, 2002). It opens a new approach to developing a formulation of IDAF. A few studies have assumed that rainfall intensity has scale invariance and used that analysis to derive scaling properties rainfall (e.g., De Michele *et al.*, 2001; Nhat *et al.*, 2007), with this approach, this study is to deal with the question how the rainfall properties at a point scale linked with areal rainfall in terms of time and space. Scaling properties of extreme rainfall in time and space explored for either disaggregation of rainfall intensity from low to high-resolution time scale or aggregation from point to area in spatial scale. Consequently, based on the scale invariance in time and space of rainfall characteristics, the IDAF curves derived.

The IDAF curves are determined for the evaluation of design rainfall using a scaling approach. The variability of annual maximum rainfall intensity in area and duration are represented through the scaling properties in time and space. Thus, the scaling relationships of mean rainfall intensity with area and duration are derived using the concepts of scaling properties. Using Extreme Value type 1 (EV1) distribution, the authors obtain a scaling model to give the IDAF curves of extreme rainfall. An application is made to the Yodo River catchment of Japan. The approach is expected to be more useful and practical to evaluate design rainfall for a specified area.

2. METHODOLOGY

2.1 Scale invariance properties of rainfall in time

Let the random variable I_d the maximum annual value of local rainfall intensity over a duration d. It is defined as:

$$I_d = \max_{0 \le t \le 1 year} \left[\frac{1}{d} \int_{t-d/2}^{t+d/2} X(\xi) d\xi \right] \quad (2)$$

where $X(\xi)$ is a time continuous stochastic process representing rainfall intensity and d is duration. It is supposed that I_d represents the Annual Maximum Rainfall Intensity (AMRI) of duration d, defined by the maximum value of moving average of width d of the continuous rainfall process. Here, some concepts are introduced about scaling of the probability distribution of random functions. A generic random function I_d is denoted by simple scaling properties if it obeys the following:

$$I_d \stackrel{dist}{=} \left(\frac{D}{d} \right)^{-H_d} I_D \quad (3)$$

D is a aggregated time duration, i.e.: 2, 3... 24 hours.

Defining the scaling ratio is $\lambda_d = \frac{D}{d}$

$$I_d \stackrel{dist}{=} \lambda_d^{-H_d} I_{\lambda_d d} \quad (4)$$

The Equation (4) is rewritten in terms of the moments of order q about the origin, denoted by $E\left[(I_d)^q \right]$. The resulting expression is:

$$E\left[(I_d)^q \right] = \lambda_d^{-H_d q} E\left[(I_{\lambda_d d})^q \right] \quad (5)$$

If one assumes the wide sense simple scaling exists, the scale-invariant models enable us to transform data from one temporal to another one, and thus, help to overcome the difficulty of inadequate. The data distribution of IDF for short-duration of rainfall intensity can be derived from daily rainfall.

2.2 Scale invariance properties of rainfall in space

Let consider a continuous precipitation process $I_{(d,a)}$ which represents the maximum annual value of average rainfall intensity over a duration d, and an area a. It is defined as

$$I_{(d,a)} = \max_{0 \le t \le 1 year} \left[\frac{1}{da} \int_{t-d/2}^{t+d/2} \int_a X(\xi, \omega) d\omega d\xi \right] \quad (6)$$

where $X(\xi, \omega)$ is a time-space continuous stochastic process representing rainfall intensity.

Since the probability of extreme events is usually examined, the maximum annual rainfall intensity can be defined as the maximum value of a moving average (with area span) in a given year. The concepts are introduced about scaling of the probability distribution of random functions. A generic random function $I_{(d,a)}$, with fixed duration d, is denoted by the simple scaling properties of space if it obeys the following relationship:

$$I_{(d,a)} \overset{dist}{=} \left(\frac{A}{a}\right)^{-H_a} I_{(d,A)} \quad (7)$$

Defining the space scaling ratio is $\lambda_a = \frac{A}{a}$

$$I_{(d,a)} \overset{dist}{=} \lambda_a^{-H_a} I_{(d,\lambda_a a)} \quad (8)$$

This also implies that the moments of any order are scale-invariance. The resulting expression is

$$E\left[(I_{(d,a)})^q\right] = \lambda_a^{-H_a q} E\left[(I_{(d,\lambda_a a)})^q\right] \quad (9)$$

where q denotes the order of the moment. If one assumes the wide sense simple scaling exists, the IDAF curves can be derived by developing a general framework based on the estimation of a common scaling exponent for any frequency level. This approach is used to derive the distribution of the IDAF where data for the spatial scale of interest does not exist.

2.3 Estimation of Intensity-Duration-Area-Frequency curves

The IDAF curves (with fixed Area) are often fitted to the extreme value type I (EVI) distribution developed by Gumbel and it is still the most often used distribution by many national meteorological services in the world to describe rainfall extremes. It will also be used in this study along with the method of moments. The annual maximum rainfall intensity $I(d)$ has a cumulative probability distribution CDF (Gumbel, 1958), which is given by

$$\begin{aligned} &F\left[I(d)\right] = 1 - \frac{1}{T} = \exp[-\exp\{-\left[I(d) - \mu\right]/\sigma\}] \\ &I(d) = \mu - \sigma \ln(-\ln T) \end{aligned} \quad (10)$$

where the location parameter μ and scale parameter σ to be calculated from data series based on L moment method .

According to the scaling theory, the IDAF formula can be derived (Nhat *et al.*, 2007) with

$$\begin{cases} I_{d,T} = \dfrac{\mu^* + \sigma^*[-\ln(-\ln(1-1/T)]}{d^{-H}} \\ \mu^* = \mu_{(\lambda d)}.(\lambda)^{-H};\sigma = \sigma_{(\lambda d)}.(\lambda)^{-H} \end{cases} \quad (11)$$

It is worthwhile to note that the simple scaling hypothesis leads to the equality between the scale factor and the exponent in the expression relating rainfall intensity and duration. The IDAF relationship can be derived from longer duration data series based on three parameters: scale exponent, the location and scale parameters of EVI distribution.

An application of the scale invariance concept in hydrology is presented for disaggregation (or downscaling) of rainfall intensity from low resolution (e.g., 1-day or 1-hour) down to high resolution (e.g., 1-hour or 10-min).

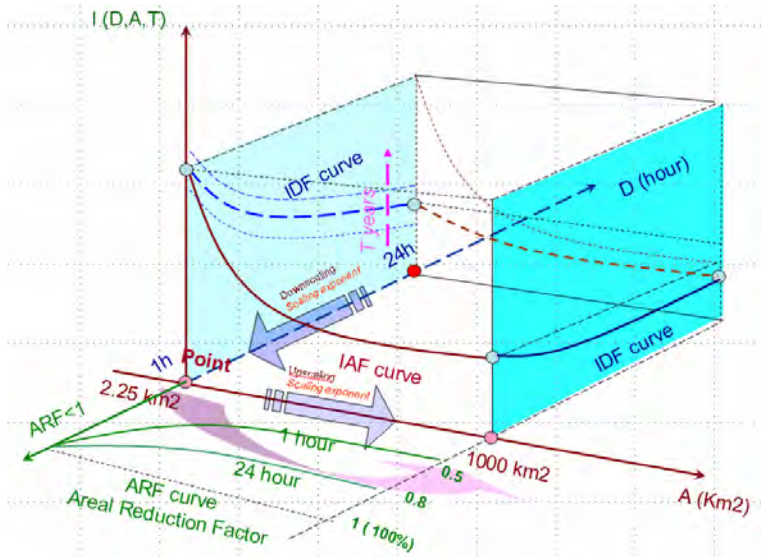

Figure 1 Derivation of intensity-duration-frequency curves and areal reduction factor.

In the other words, statistical properties of rainfall for short durations can be inferred form those of rainfall data available for longer duration. In principle, the statistical properties (e.g., moments) of the scaling relationship could be obtained from large scales and then used to estimate the process properties at smaller scales as will be illustrated in the derivation of scaling IDAF curves.

The IDAF curves and ARF reflect the variability of rainfall in time and space, thus it is necessary to make joint analysis of scaling properties of the rainfall field in duration and area (Figure 1)

3. APPLICATION

For time scaling, the simple scaling theory can be applied to derive IDAF curves consistent with hourly rainfall series in rain gauges where only daily data are available. These curves are developed for gauged sites based on scaling of the generalized extreme value (GEV) and Gumbel probability distributions. Statistical analysis was performed on annual maximum rainfall series for the Yodo River catchment for durations ranging from 1 hour to 24 hours. The results showed that rainfall does follow a simple scaling process in time (Nhat *et al.*, 2007). To investigate the scaling invariance behaviour of the spatial rainfall representation, analyses of the statistical moment are made, similar as for the time series. At first hourly data from 1982 to 2002 with 1.5 x 1.5 km spatial resolution, which is generated by AMeDAS rain gauge data and covers the Yodo River catchment are arranged. To obtain spatial mean rainfall intensity, the Otorii station is put at the center of the area, and the area is expanded in the form of circle with increased radius by 0.5 km (see Figure 2). The data used for examined spatial scaling factor are annual maximum rainfall series for the area from 2.25 km^2 to 5150.25 km^2, with λ_a =1, 5...2289 and fixed duration d. As previously mentioned, one of these describes the variation of statistical moments with scale as

$$E\left[(I_{(d,a)})^q\right] = \lambda_a^{K(q)} E\left[(I_{(d,\lambda_a a)})^q\right] \qquad (12)$$

where $E\left[(I_{(d,a)})^q\right]$ is the ensemble average qth moment of the field studied at (i.e. averaged over) a scale specified by scale ratio λ_a.

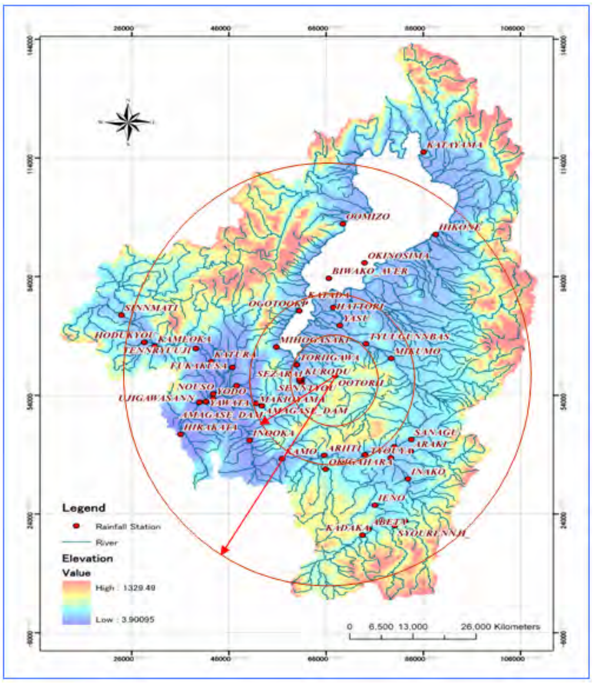

Figure 2 Spatial mean area rainfall intensity expanded in the form of circle.

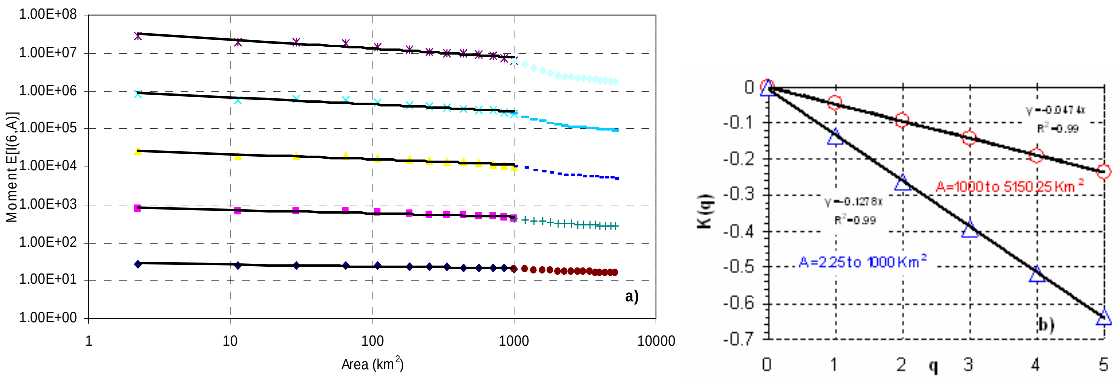

Figure 3 The space scale invariance at the Yodo River catchment for 6 hours.
a) Relationship between moment of order q and area A (km^2).
b) Relationship between K(q) and the sample moment order q.

Figure 3a show the q^{th} statistical moment, $E\left[(I_{(d,a)})^q\right]$, fixed d=6hour, as function of λ_a for one typical example of each investigated rainfall scale invariance. Results are shown for order of q=1, 2, 3, 4, and 5. The straight-line behavior is well respected between approximately λ_a=1 (A =2.25 km^2) and λ_a= 2889 (A~ 5000 km^2), with mean R^2 of the fitted regression lines for all q equal to 0.99 (0.96-1.0).

In Figure 3b, the difference of the degree of steepness in the slopes for the area (2.25 km^2 to 1000 km^2) and larger area (1000 km^2 to 5000 km^2) are found, which indicates that two different scaling space regimes exist for rainfall space scaling. A steeper slope is found in the smaller area compared to the lager area. The plots indicate that the relationships between moments and areas are linear having two different slopes with a breaking point at 1000 km^2. This property suggests an existence of the two different regimes with a transition in storm dynamics from high variability convective storms with less than 1000 km^2 to frontal storms in an area larger than 1000 km^2. The rainfall intensity-duration-area-frequency (IDAF) can be derived from a small area to a larger area based on space scaling present in Figure 4.

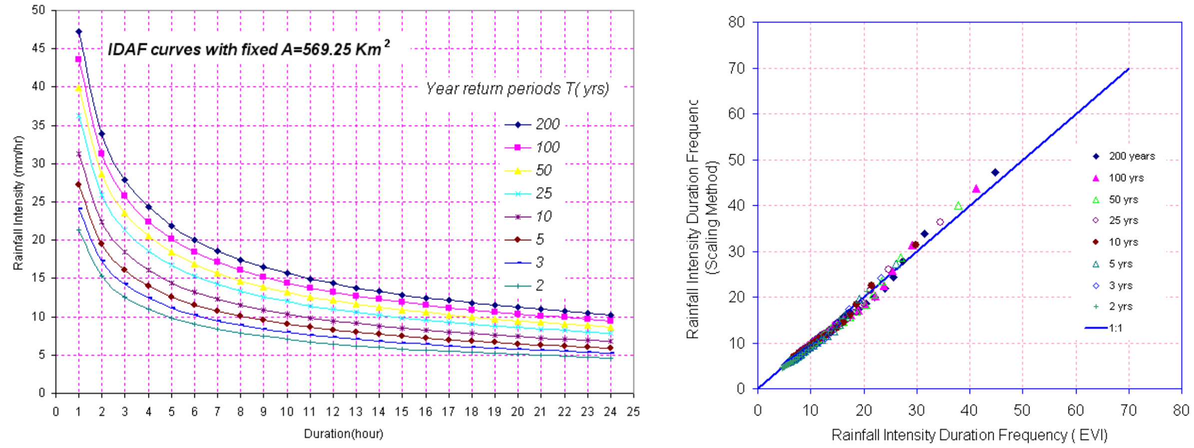

Figure 4 The IADF curves constructed by the scaling method and comparison with traditional method for the A=569 km^2

For comparison purposes, empirical IDAF curves for AMRI rainfalls are also constructed using the traditional method by which the extreme rainfall quantiles were estimated for each duration independently using the observed rainfall data for that particular duration and area; that is, without considering the scaling properties of rainfall series for other durations. Figure 4 show the IDAF curves constructed by the scaling method approach together with those given by traditional method for the A=569 km^2. It can be seen that both methods produce IDAF curves that are similar in terms of both the shape and rainfall intensity estimates.

Hence, it can be concluded that the proposed scaling method could provide IDAF curves that are comparable to those derived by traditional method. However, the scaling approach was found to be more efficient since it can provide extreme rainfall estimates for all rainfall duration simultaneously and using a smaller number of parameters as well, while the traditional method requires the estimation of rainfall estimates for each duration, independently. Another interesting feature of the proposed scaling procedure is its ability to derive IDAF curves for rainfall durations that have not been observed or measured. The traditional method can not be used for cases without data.

We adopt a statistical analysis to obtain the Areal Reduction Factor (ARFs) based on its scaling properties in space and time. The concepts of the statistical scaling are used to study the variability of a random process in time and space. The approach is expected to be more useful and practical to evaluate design rainfall for a specified area. The figure 5 shows ARF against some of most important empirical ARF-are relationship found in the literature.

$$\begin{aligned}
&\text{- } \textit{Leclerc - Schaake } (1972) \\
&ARF = 1 - \exp(-1.1 d^{0.25}) + \exp(-1.1 d^{0.25} - 0.003863 A) \\
&\text{- } \textit{Horton's Relationship } (2003) \\
&ARF = \exp(-0.09 * A^{0.23})
\end{aligned} \tag{13}$$

It is evident that all of the empirical curves underestimate the ARF for large areas (*A*) and higher duration (*D*). Also, it is possible to note that empirical curves overestimate the ARF for small area (*A*).

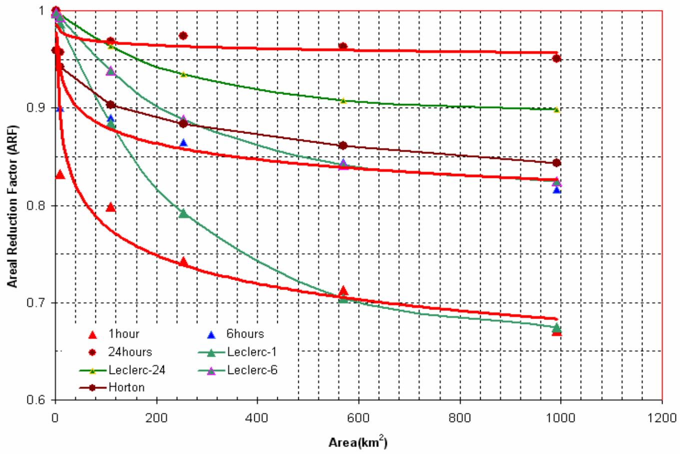

Figure 5 The areal reduction factor derived based on the scale invariance of rainfall

4. CONCLUSION

The properties of scale invariance of rainfall quantiles are examined in the Yodo River catchment: For space scaling, a simple scaling process with the two different regimes for areas less than 1000 km^2 and more than 1000 km^2 are found. For time scaling with fixed A, rainfall properties follow a simple scaling. The rainfall IDAF curves for short duration (hourly) were derived from daily rainfall data. The IDAF relationships are deduced from daily rainfall, which show good results in comparison with IDAF curves obtained from at-site short-duration rainfall data. The Areal Reduction Factor (ARFs) was derived based on scaling properties of rainfall.

Results of this study are of significant practical importance because statistical rainfall inferences can be made with the use of a simple scaling in time and space. Furthermore, daily data are more widely available from standard rain gauge measurements, but data for shorter durations are often not available for the required site.

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REFERENCES

Asquith, W.H. and Famiglietti, J.S. (2000), Precipitation areal-reduction factor estimation using an annual-maxima centered approach, *Journal of Hydrology*, 230, pp.55-69.

Bacchi B. and Ranzi R. (1996), On the derivation of the areal reduction factor of storms, *Atmospheric Research*, 42, pp.123-135.

De Michele, C., Kottegoda, N.T., and Rosso, R. (2001), The derivation of areal reduction factor of storm rainfall from its scaling properties, *Water Resources Research*, 37(12), pp.3247-3252.

Gupta, V. K. and Waymire, E. (1990), Multiscaling properties of spatial rainfall and river flow distributions, *Journal of Geophysical Research*, 95(D3), pp.1999-2009.

Nguyen, V. T. V., Nguyen, T.-D., Ashkar, F. (2002): Regional frequency analysis of extreme rainfalls, *Water Sciences and Technology*, 45(2), pp.75–81.

Nhat, L. M., Tachikawa, Y., Sayama, T., and Takara, K. (2007), A simple scaling characteristic of rainfall in time and space to derive intensity duration frequency relationships, *Annual Journal of Hydraulic Engineering*, JSCE, Vol.51, pp.73-78.

Omolayo, A.S. (1993), On the transposition of areal reduction factors for rainfall frequency estimation, *Journal of Hydrology*, 145, pp.191-205.

Rodriguez-Iturbe, I. & Mejia, J.M. (1974), On the transformation of point rainfall to areal rainfall, *Water Resources Research*, 10(4), pp.729-735.

Sivapalan, M., Blöschl G. (1998), Transformation of point rainfall to the areal rainfall: Intensity-duration-frequency curves, *Journal of Hydrology*, 204, pp.150-167.