EXTENDING A STORAGE-DISCHARGE RELATIONSHIP FOR SUBSURFACE FLOW MODELING IN DRY MILD-SLOPE BASINS

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In this study, a new storage-discharge equation for unsaturated zone considering the soil moisture suction is proposed for dry mild-slope basins. For wet steep basins a distributed hydrologic model with kinematic wave approximations including surface-subsurface runoff, OHDIS-kwmss has been well studied. However, when we apply this model for a dry mild-slope basin, the Illinois basin (2,400 km^2) in U.S, we found that the model predictions were not good. Especially, we observed that the model was not capable to reproduce the hydrograph well even for the calibrated event itself by parameter tuning . Soil moisture suction effect is important in dry mild-slope basins and the present equation does not include the effect of soil moisture suction. Therefore, we derive a new equation by including the soil moisture suction effect. Finally, the new equation is tested for different hillslopes and it is found that the new equation shows different flow characteristics from the existing equation.

Key Words: *subsurface flow, distributed hydrological model, storage-discharge relationship, mild- slope basins, OHDIS-kwmss, prediction in ungauged basins*

1. INTRODUCTION

Understanding the governing hydrological processes in different regions of the world is necessary to realize predictions in ungauged basins. In this regards, physically realistic models can play a major role. However, in practice simplicity of the process is also needed because even at hillslope scales complicated models such as three dimensional Richards equation are not feasible[1]. Furthermore, measuring physical properties of soils, vegetation etc. in finer spatial and temporal resolutions is not realistic. Therefore, lumping processes in some extent is necessary for basin scale applications. Kinematic approximation is one of the simplified approaches, which is used to model the hydrological process in both surface and subsurface flow[2),3),4),5)].

OHDIS-kwmss is a distributed hydrological model with kinematic wave approximations including surface-subsurface runoff developed at Kyoto University[3),6)]. In general, Japanese basins are wet steep and this model has been successfully applied for many basins in Japan. Hunukumbura *et al.*[7)] applied the model to the Maruyama River basin (909 km^2) in Japan and found that the model calibrated for small flood event can be used to predict large flood events as well with good prediction accuracy. Further they concluded that the calibrated model using the runoff data at the basin outlet is capable of predicting the inside locations with reasonable accuracy. However, this model is not tested in other places in the world. Applications of hydrological models and testing their capabilities in different basins in the world are vital to understand the governing hydrological processes in each region.

In this study, we investigate the prediction capability of OHDIS-kwmss model for the Illinois River basin in U.S, which is one of the experimental basins of the Distributed Model Inter-comparison Project, DMIP [8),9)]. In DMIP phase 1, seven distributed hydrological models (SWAT, NOAH Land Surface model, HRCDHM, HL-RMS, VIC-

3L, TOPNET and WATFLOOD) were applied to the Illinois basin at Tahlequah. Results show that the overall Nash coefficient for each model varies from 0.27 to 0.85 for the calibration period. Event absolute error for all the models for the same period is greater than 20%. The best Nash-coefficient value for the calibration period (0.85) was obtained from the HL-RMS model with Sacramento Soil Moisture Accounting Model (SAC-SMA), and hillslope-channel routing employs the kinematic wave. This model uses larger spatial resolution (16 km^2) grids [10),11),12)].

Our experience in OHDIS-kwmss in wet steep basins in Japan shows higher Nash-coefficient value (greater than 0.95) for model calibration period. As the model was not tested yet in dry mild-slope basins, main objectives in this study are to check whether the present model structure is good enough to capture the hydrological process in dry mild-slope basin and if it is not sufficient, which modification is necessary to make good prediction in dry mild-slope basins.

The model is calibrated for the basin by using SCE-UA algorithm[13),14)] and after applying the calibrated model parameters for other events, it is observed that the model predictions are not good in the Illinois River basin. Even for the calibrated event, it was not possible to reproduce the observed outflow hydrograph well by parameter tuning. It means that the present model equations representing the surface and subsurface flow with kinematic wave approximation, is not sufficient to capture the governing hydrological processes in the basin. The Illinois River basin is mild-slope dry basin, and hence the effect of soil moisture suction in the unsaturated zone would be significant.

In this study a new storage-discharge equation for unsaturated zone considering the soil moisture suction is proposed. The new equation is based on the Darcy's equation, and soil moisture suction head term is replaced using Campbell's simplified model[15)] representing the soil moisture and hydraulic conductivity characteristics. The equation is possible to include the effect of soil moisture suction by adding only one additional parameter to the existing model. Finally, the new storage-discharge equation is tested for single hill slopes with different parameters and the slope flow characteristics of the new equation is compared with the existing equation.

Section 2 describes the existing model used in this study and its applications to the Illinois basin. Section 3 presents the derivation of the new equation and numerical solution. Application and comparison of the existing and new equation are given in section 4.

2. EXISTING MODEL APPLICATIONS

(1) The existing storage-discharge equation

OHDIS-kwmss is based on the one dimensional kinematic wave theory and developed by Ichikawa *et al.*[6)]. The basin topography in the hydrologic model is represented according to the methodology described in Shiiba *et al.*[16)]. In the distributed hydrologic model, it is considered that the basin consists of number of rectangular slope elements which drains to the deepest gradient of its surrounding.

It is assumed that the each slope element of the basin is covered with a permeable soil layer and a storage-discharge relationship for unsaturated, saturated and overland flow defined by the threshold d_m and d_a[3)] is adopted. The stage discharge relationship used in the OHDIS-kwmss model are given in Eq.(1). In this equation, the soil moisture suction head in the unsaturated zone is not included. Therefore, it is supposed that the equation is basically applicable for hill-slopes with steep slope angle and wet condition:

$$q = \begin{cases} k_m d_m i (h/d_m)^{\beta} & 0 \le h \le d_m \\ k_m d_m i + k_a i (h - d_m) & d_m < h \le d_a \\ k_m d_m i + k_a i (h - d_m) + \alpha (h - d_a)^m & d_a < h \end{cases} \quad (1)$$

where, q is the discharge per unit width; h is the stage; i is the slope; k_m and k_a are saturated hydraulic conductivities in the capillary pore and non-capillary pore respectively.

(2) Application of OHDIS-kwmss model for the Illinois River basin

To check the capability of the present storage-discharge equation to capture the hydrological processes in different type of basins, we have applied the model to the Illinois River basin in US (**Fig. 1).** It is one of the Distributed Model Inter-comparison Project experimental basins[8),9)]. The Illinois basin at Tahlequah is about 2,400 km^2 in size and situated in both Oklahoma and Arkansas state, U.S. The average annual flow rate at the basin outlet and the average annual rainfall of the basin are about 29 m^3/s and 1142 mm, respectively. In the basin, rocky soils and outcrops tend to remain either forest or pasture[12)]. Hourly precipitation data derived from gauged-adjusted radar and hourly stream flow data at Tahlequah are obtained from DMIP website[9)]. Altogether 15 flood events occurred between 1996 to 2001 were selected for this study. The summery of all selected events are given in Table 1. The 30 m digital elevation model is also downloaded from the same website and it was resampled to 300 m grid size. The OHDIS-kwmss model was then set up to the basin.

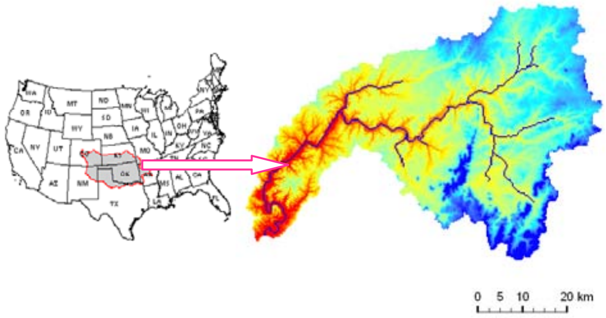

Fig. 1 The Illinois River basin.

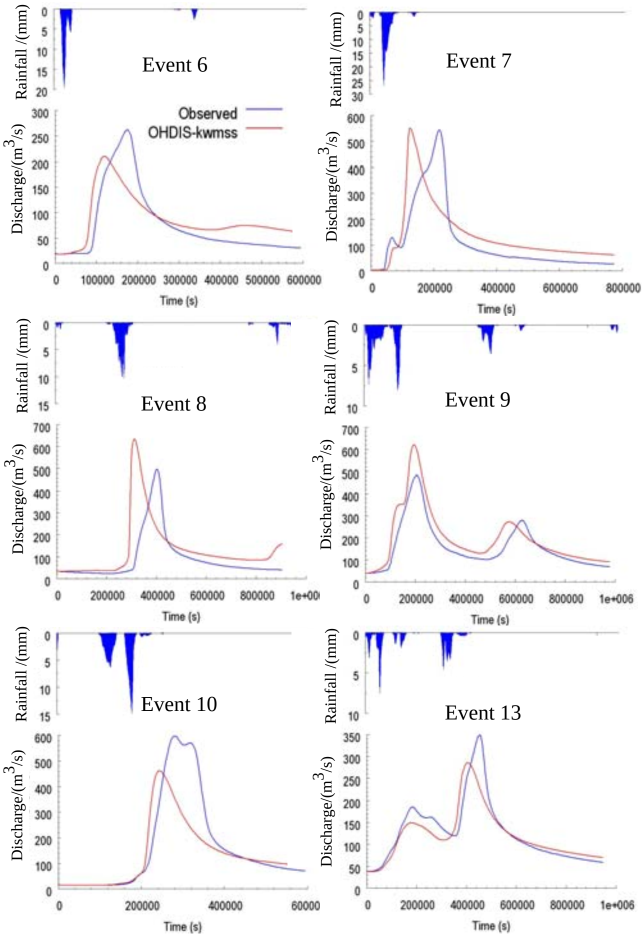

Fig. 2 Model calibration and validation at the Illinois River basin.

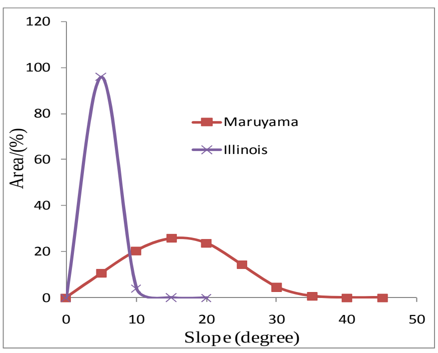

Fig. 3 Slope-area comparison for the Maruyama and the Illinois River basins.

Table 1 Detail of the selected flood events at Tahlequah.

DMIP ID	Total RF (mm)	Total Flow (mm)	Initial Flow (mm)
Event-6	58	18.2	0.026
Event-7	120	35.1	0.005
Event-8	92	32.9	0.049
Event-9	110	63.2	0.059
Event-10	87	38.9	0.020
Event-11	75	5.0	0.011
Event-12	135	81.6	0.041
Event-13	67	48.5	0.054
Event-14	105	17.0	0.007
Event-15	37	28.4	0.043
Event-16	15	17.4	0.048
Event-17	95	35.8	0.046
Event-18	108	48.5	0.051
Event-19	39	5.8	0.011
Event-20	56	14.4	0.013

(3) Results and discussion for Illinois River basin

The model was calibrated for the flood Event 6 in **Table 1** using SCE-UA algorithm and the calibrated model parameters were then use to predict the other 14 events. Prediction results of some events including the calibration event are shown in **Fig. 2**.

According to our past experience in Japan, calibrated parameters for any event can be used to predict the other events with good prediction accuracy[7)]. In contrast, the model predictions are not good for all events in Illinois basin (**Fig.2**). Therefore, it is clear that the calibrated parameters for Event 6 are not suitable to predict the other events. For further clarification on this, we have calibrated the model using Event 9 data and used the calibrated parameters to reproduce other events. We observed similar results. It means that, the existing storage discharge relationship is not well capture the governing hydrological process in the Illinois basin. Furthermore, it is found that the objective function value (Nash coefficient) for model calibration is 0.68 and it is much lower value than that of usually experienced in Japanese basins. Moreover, Event 9 and 13 shows little better prediction than others. From **Table 1**, it can be noted that the initial flow rate of the Event 9 and 13 are higher than that of other events. In other words, initial condition of the basin for these two events is wetter than the other events. Therefore, it is concluded that the hydrological responses of this basin are highly nonlinear for dry conditions.

Considering all these facts and figures, the model is not applicable to reproduce the observed hydrographs even for calibrated events by changing the model parameters. Thus we concluded that the present storage-discharge equation is not sufficient for flood prediction of dry and mild-slope basins such as the Illinois basin. **Figure 3** shows the slope-area distributions of the Illinois basin and the Maruyama basin in Japan. According to this, the average slope of the Illinois basin is about 5 deg while that of the Maruyama basin is about 20 deg.

In the derivation of the stage discharge equation for the unsaturated zone, kinematic wave approximations were used in the present model and the soil moisture suction head term was neglected assuming the steep slope. Illinois basin is a dry mild-slope basin. Therefore the effect of soil moisture suction head is important. We believe that the extension for the present model equations considering the effect of soil moisture suction head is necessary for modeling highly nonlinear dry mild-slope basins.

3. DERIVATION OF NEW STORAGE-DISCHARGE EQUATION FOR UNSATURATED ZONE

(1) Derivation of new storage-discharge equation

The subsurface flow is conceptualized as two types of flows, saturated flow through non-capillary pores and unsaturated flow through capillary pores. First, unsaturated flow happens and after the water storage in the soil becomes greater than a threshold d_m, the saturated flow starts. **Figure 4** shows the vertical section of the hill-slope, and **Figure 5** shows the water storage h profile along the slope. The total pore volume, capillary pore volume and water storage in the soil at each cross section of the slope are represented as height d_a, d_m and h.

For the unsaturated flow ($0 \leq h < d_m$), by applying the Darcy's equation along the slope, average moisture flux velocity v_m to the down slope direction can be written as;

$$v_m = -k\frac{\partial H}{\partial x} = -k\frac{\partial(z+\varphi)}{\partial x} = k\left(i - \frac{\partial \varphi}{\partial x}\right) \tag{3}$$

where k, i, H, z and φ, hydraulic conductivity, slope, total head, elevation head and the moisture suction head respectively.

In the case of steep, the difference of the soil moisture suction head in the flow direction can be assumed very small as compared to the difference of the elevation head. Therefore the suction head term can be neglected. However, for mild slope this could significant and should be included. To incorporate the soil moisture suction term, Campbell's model (Eq.4) for soil moisture and hydraulic conductivity is used:

$$\frac{\alpha}{\phi} = \left(\frac{H_a}{\varphi}\right)^{1/b} \quad and \quad \frac{k}{k_m} = \left(\frac{\alpha}{\phi}\right)^{\beta} \tag{4}$$

where, k_m, ϕ, α and H_a are saturated hydraulic conductivity, porosity, moisture content and air entry suction pressure respectively ($H_a < 0$). The parameter b is a constant and defines the soil

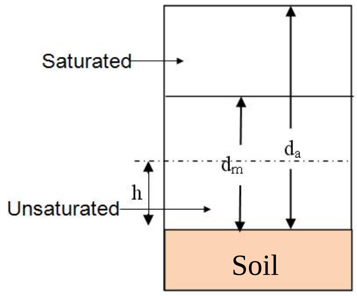

Fig. 4 Cross section of the soil layer.

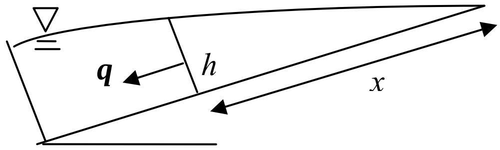

Fig. 5 Soil water storage variation along the slope.

moisture suction curve. The parameter β has relationship of $\beta = 3+2b$.

According to the setting shown in **Fig. 4**,

$$\frac{\alpha}{\phi} = \frac{h}{d_m}$$

Therefore;

$$\varphi = H_a\left(\frac{h}{d_m}\right)^{-b} \quad and\ k = k_m\left(\frac{h}{d_m}\right)^{\beta} \tag{5}$$

Using Eq.3 and Eq.5 we can get;

$$v_m = k\left(i + bH_a\frac{1}{d_m}\left(\frac{h}{d_m}\right)^{(-b-1)}\frac{\partial h}{\partial x}\right) \tag{6}$$

Therefore, the moisture flux rate per unit width, q can be written as a function of storage and its gradient;

$$q = k_m d_m i\left(\frac{h}{d_m}\right)^{\beta} + k_m bH_a\left(\frac{h}{d_m}\right)^{(\beta-b-1)}\frac{\partial h}{\partial x} \tag{7}$$

The difference between this new equation and the existing equation for the unsaturated zone is that the new equation has an additional term with the storage gradient, the second term of Eq.7. Interestingly, it is possible to incorporate the effect of soil moisture suction by using only one additional parameter H_a to the existing equation. Furthermore, by setting the parameter H_a to zero in the Eq.7, we can obtain exactly the same equation as in the current OHDIS-kwmss model. Continuity equation of the water flow in the slope element can be written as:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t) \tag{8}$$

where $r(t)$ is the rainfall intensity to the slope element.

Finally substituting the Eq.7 to Eq.8 we can obtain;

$$\frac{\partial h}{\partial t}+A\frac{\partial h}{\partial x}+B\left(\frac{\partial h}{\partial x}\right)^2+C\frac{\partial^2 h}{\partial x^2}=r(t) \tag{9}$$

where;

$$A=k_m i\beta\left(\frac{h}{d_m}\right)^{(\beta-1)},\ B=\frac{k_m bH_a(\beta-b-1)}{d_m}\left(\frac{h}{d_m}\right)^{(\beta-b-2)}$$

$$C=k_m bH_a\left(\frac{h}{d_m}\right)^{(\beta-b-1)}$$

(2) Numerical solution

This non-linear equation (Eq.9) needs to be solved for h to calculate q. In this regard, A, B and C in the Eq.9 are calculated using known time level while the derivatives are calculated implicitly[17]. Following boundary conditions are assumed to solve the above equation.

For $x = 0$; $h = 0$ for all t and

For $t = 0$; $h = 0$ for all x

Using the five point implicit scheme, partial derivatives can be written as;

$$\frac{\partial h}{\partial x}=(1-\theta)\frac{h_{n-1,j}-h_{n-1,j-1}}{\Delta x}+\theta\frac{h_{n,j}-h_{n,j-1}}{\Delta x}$$

$$\frac{\partial h}{\partial t}=(1-\lambda)\frac{h_{n,j-1}-h_{n-1,j-1}}{\Delta t}+\lambda\frac{h_{n,j}-h_{n-1,j}}{\Delta t}$$

$$\frac{\partial^2 h}{\partial x^2}=(1-\theta)\frac{h_{n-1,j}-2h_{n-1,j-1}+h_{n-1,j-2}}{\Delta x^2}+\theta\frac{h_{n,j}-2h_{n,j-1}+h_{n,j-2}}{\Delta x^2}$$

Substituting above derivatives into Eq.9, h is represented as:

$$h_{n,j}=\frac{\Delta t}{\lambda}\left\{\begin{aligned}&r(e)-A\left[\frac{\theta(h_{n,j}-h_{n,j-1})+(1-\theta)(h_{n-1,j}-h_{n-1,j-1})}{\Delta x}\right]\\&-B\left(\frac{\theta(h_{n,j}-h_{n,j-1})+(1-\theta)(h_{n-1,j}-h_{n-1,j-1})}{\Delta x}\right)^2\\&-C\left[\frac{\theta(h_{n,j}-2h_{n,j-1}+h_{n,j-2})}{\Delta x^2}+\frac{(1-\theta)(h_{n-1,j}-2h_{n-1,j-1}+h^{n}_{n-1,j-2})}{\Delta x^2}\right]\end{aligned}\right\} \tag{10}$$

$$-\frac{(1-\lambda)}{\lambda}\left(h^{1}_{n,j-1}-h_{n-1,j-1}\right)+h_{n-1,j}$$

where, n and j are current time step and current spatial step, respectively. The parameters λ and θ are weights factors. We use 1 and 0.5 for λ and θ respectively. Eq.10 is solved for $h_{n,j}$ iteratively and then find the corresponding the value of q.

4. APPLICATION AND COMPARISION OF THE NEW EQUATION WITH EXISTING EQUATION

One simple hillslope is used to study the characteristics of the new equation and to compare it with the existing equation. The same parameter values were select for both models except the H_a parameter which is the additional parameter used in the new equation. The parameter values are given in **Table 2**. It is assumed that the hill-slope is initially fully dried. Constant rainfall intensity (60mm/hr for 10 hours) is applied for the hillslope and two cases are tested; 1) rainfall is given to the entire hillslope, (this represent slope elements near to the river or edges with more water content in the downstream) (**Fig. 6**); and 2) to the upper half of the slope (this represent when the rain occurred on hill mountains while the downstream is dry) (**Fig. 7**) to understand the characteristics of flow for spatially distributed rainfall. Outflow hydrographs are simulated for both equations for different slope angles. With the parameters which we have chosen, the soil remains unsaturated throughout the simulation period.

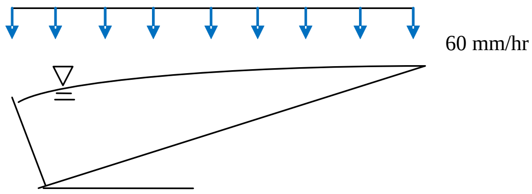

Fig. 6 Test case 1 – rainfall applied to the entire hillslope.

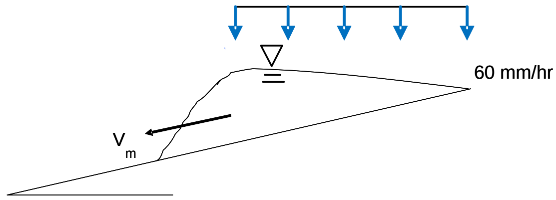

Fig. 7 Test case 2 – rainfall applied at the upper half.

Table 2 Parameter values used in the models.

Parameter	value
K_m (cm/hr)	0.05
β (-)	4
d_m (mm)	600
d_a (mm)	650
H_a (mm)	-100
Slope Length (m)	100

Figures 8 and **9** show the simulated hydrographs of the new and existing equations for different hillslope angles for Case 1 and Case 2, respectively. When apply the rainfall for entire slope (Case 1), it is found that difference of the simulated hydrographs from new and existing equation is not significant. It means if the soil moisture distribution pattern looks like **Fig.6**, the differences of discharges by the two equations are small. In Case 2, for higher slope angles, the difference between two equations is also not significant. Even though we consider the soil moisture suction in the new equation, we cannot see difference because the elevation head difference for steep slopes is considerably larger than the difference of soil moisture suction. In contrast, we can see a clear

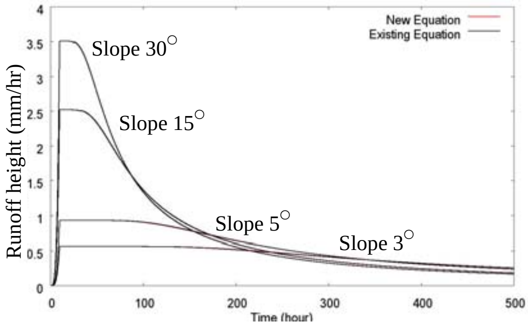

Fig. 8 Test case 1 – Simulated hydrographs for different slope angles.

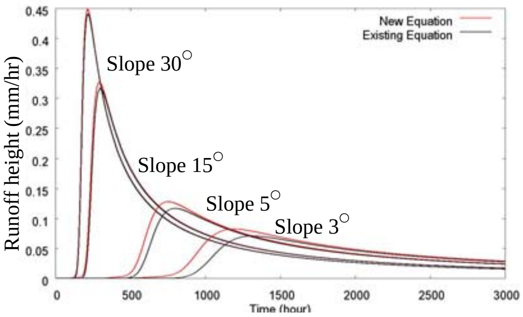

Fig. 9 Test case 2 – Simulated hydrographs for different slope angles.

difference of the simulated hydrographs for mild slopes in Case 2. For mild slopes, when the down slope is dry, the effect of soil moisture suction becomes significant and the kinematics approximation is inapplicable. In general, when the down slope soil is dry, moisture flux velocity tends to have higher value due to the effect of soil moisture suction. The new equation captures this phenomenon and starts the flow at the end of the slope earlier than that of the present equation.

The higher velocity of unsaturated flow in the new equation leads to identify smaller hydraulic conductivity. This means if heavy rainfall is provided, the soil is easily saturated and large flood will happen. We believe the high nonlinear characteristics of the new equation will provide better simulation results at the dry mild-slopes basins.

5. CONCLUSIONS

- Even though the existing storage discharge equation is capable of giving good predictions for wet and steep basins, flood predictions in dry mild-sloped basins were not good.
- New storage discharge relationship for unsaturated zone considering the soil moisture suction was developed.
- Simulated flow by using both equations for hillslopes with different slope angles were obtained and compared. For steep slopes, both models give nearly the same results. However for mild slopes with dry soil in the down slope, it is found that the effect of lateral soil moisture suction difference has significant effect on the discharge.

Further research is to apply the distributed hydrologic model with the new storage-discharge equation to the Illinois River basin and confirm the applicability of the new equation for dry and mild-slope basins.

REFERENCES

1) Troch, P. A., Loon, E.E.V and Hilberts, A.G.J: Hillslope-storage Boussnessq model for subsurface flow and variable sources area along complex hillslopes:2Intercomparison with a three-dimentional Rechards equation model, *Water Resours. Res.*, Vol.39, No.11, 2003.
2) Beven,K. : On subsurface stromflow: predictions with simple kinematic theory for saturated and unsaturated flows, *Water Resours. Res.*, Vol.18, No.6, 1982.
3) Tachikawa, Y., Nagatani, G. and Takara, K: Development of stage-discharge relationship equation incorporating saturated–unsaturated flow mechanism, Ann. J. Hydraulic Engng. JSCE 48, 7-12, 2004.
4) Troch, P. A., Loon, E.V and Hilberts, A: Analytical solution to hillslope-storage kinematic wave equation for subsurface flow, Advance in Water Resources, 25, 637-649, 2002.
5) Ichikawa, Y., Murakami, M., Tachikawa, T., and Shiiba, M: Development of a basin runoff simulation system based on a new digital topographic model, J. Hydraulic, Coastal and Environ. Engng. JSCE 691 (II-57)43-52, 2001.
6) Shiiba, M., Tachikawa, T., Ichikawa, Y., Hori, T, and Tanaka K: Development of a slope runoff models which consider field capacity and pipe flows, Annuals of Disas. Prev. Res. Inst., Kyoto Univ., No. 41B-2, 1998.
7) Hunukumbura, P.B., Tachikawa, Y. and Takara, K: Improvement of internal behavior in distributed hydrological model, Annual Journal of Hydraulic Engineering, JSCE, Vol.52, 2008.
8) NOAA: Distributed Model Inter-comparison Project http://www.nws.noaa.gov/oh/hrl/dmip/update_log.html, January, 2008.
9) NOAA: Distributed Model Inter-comparison Project http://www.nws.noaa.gov/oh/hrl/dmip/2/index.html, February, 2008.
10) Reed, S, Koren, V, Smith, M, Zahang, Z, Moreda, F and Seo, D : Overall distributed model intercomparison project, *J. Hydrol.*, Vol.298, 27-60, 2004.
11) Koren, V, Reed, S, Smith, M, Zahang, Z, and Seo, D : Hydrology laboratory research modeling system (HL-RMS)of the US national weather service, *J. Hydrol.*, Vol.291, 297-318, 2004.
12) Vieux, E.B, Cui, Z and Gaur, A: Evaluation of a distributed hydrological model for flood forecasting, *J. Hydrol.*, Vol.298, 155-177, 2004.
13) Duan, Q., Sorooshian, S. and Gupta, V.K: Effective and efficient global optimization for conceptual rainfall-runoff models, *Water Resours. Res.*, Vol.28, No.4, 1015-1031, 1992.
14) Duan, Q., Sorooshian, S. and Gupta, V.K: Optimal use of the SCE-UA global optimization method for calibrating watershed models, *J. Hydrol.*, Vol.158, 265-284, 1994.
15) Maidment, D.R: Handbook of Hydrology, McGRAW-HILL, INC, 5.6-5.7, 1993.
16) Shiiba, M., Ichikawa, Y., Sakakibara, T. and Tachikawa, Y: A new numerical representation form of basin topography, J. Hydraulic, Coastal and Environ. Engng. JSCE 621 (II-47) 1-9,1999.
17) Singh, V.P: Kinematic wave modeling in water resources, John Wiley & Sons, 691-965, 1996.

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