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NEYMAN-SCOTT RAINFALL MODEL PARAMETERS AS INDICATORS OF TEMPORAL CHANGE IN HISTORICAL RAINFALL

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The Neyman-Scott clustered Poisson rectangular pulse rainfall model (NSM) parameters are estimated to investigate temporal change in rainfall characteristics in a historical record. This was conducted on partial rainfall series involving monthly pooled rainfall. The historical data used is based on Kyoto in Japan, where an hourly historical record from 1950 to 2002 was obtained. Each monthly record is further subdivided into four overlapping 20-year sub-records prior to parameter estimation via optimization. The method of moments is used to identify the NSM model parameters. Results indicate that storm rainfall volume in the month of September has increased throughout the 50 year historical record. Frontal rainfall in June also appears to have increased throughout the record period as well.

Key Words: *Point processes, Neyman-Scott rainfall model, climate change, rainfall temporal change*

1. INTRODUCTION

The detection of temporal change of rainfall properties has been a key issue in topics related to global climate change. Though spatio-temporal analysis of historical data offers more detailed information, the temporal analysis at a single site is sufficient in cases when regional distributions are reasonably homogeneous and/or when overall computation time is to be limited. Our motivation here is based on the need to establish evidence of rainfall temporal change that is based on a stochastic approach. We propose a methodology that can be applied systematically throughout a historical rainfall record that yields quantitative interval based volumetric measures of a storm. This in turn can be used as a standard check on the output of a synthetic rainfall generating scheme (i.e., rainfall from modeling techniques) such that the overall performance of the scheme can be assessed.

This paper focuses on using the parameters of the temporal Neyman-Scott clustered Poisson rectangular pulse rainfall model (NSM) to contribute such interval based measures. Originally proposed by Rodriguez-Iturbe *et al.*1), the NSM treats rainfall as a temporal point process2) of rain cells with properties such as intensity, duration, and arrival that can be systematically handled as random processes. The parameters of the NSM represent the probabilistic distributions of rain cell intensity, rain cell duration, rain cell arrival time and so on. We show that these parameters in turn lead to important storm characteristics. The determination of these characteristics based on the method proposed here is less arbitrary than similar methods based on historical data. Through the NSM model parameters, it is plausible to identify trends that are not easy to detect by analyzing historical rainfall time series data alone.

Historical rainfall at Kyoto used here was observed by the Automated Meteorological Data Acquisition System (AMeDAS) from 1976-2002 as well as a paper record from 1950-1976. Each monthly record is further subdivided into four overlapping 20-year sub-records prior to the NSM parameter estimation via optimization. Measures based on the parameter values are then analyzed. A brief description of the NSM appears in Section 2 followed by the parameter estimation in Section 3. Trends and other results are examined in Section 4 followed by concluding remarks in Section 5.

2. MODEL DESCRIPTION

(1) Governing equations

Rainfall in the NSM[1] is formulated as a temporal cluster of rain cells (storms) which occur based on a Poisson process with occurrence rate λ. The cluster size is based on a geometric distribution with mean μ_c. The lag from cluster to rain cell occurrences follows an exponential distribution with mean 1/β. Rain cell durations follow an exponential distribution as well with mean 1/η. Rain cell intensities are modeled by a gamma distribution with scale parameter θ and shape parameter α. The resulting rainfall is the superpositions of these processes as shown in **Fig. 1**.

The moment and correlation expressions for the aggregated rainfall point process in terms of the NSM parameters were derived by Rodriguez-Iturbe *et al.*[3] The succeeding Equations (1) – (3) list these expressions in the aggregated form, specific to the distribution parameters mentioned: β, λ, μ_c, η, α, and θ.

$$\mathbf{E}\left\langle \mathrm{Y}_n^{(h)} \right\rangle = \lambda\mu_c(\alpha\theta)h/\eta \tag{1}$$

$$\begin{aligned}\mathrm{var}\left\langle \mathrm{Y}_n^{(h)} \right\rangle = {} & \frac{2\lambda\mu_c\alpha\theta^2(1+\alpha)(\mu_c-1)}{\beta\eta^3(\beta^2-\eta^2)}\times \\ & \left[\beta^3 A_1(h)-\eta^3 B_1(h)\right]+\frac{4\lambda\mu_c(\alpha\theta)^2 A_1(h)}{\eta^3}\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{cor}\left\langle \mathrm{Y}_n^{(h)},\mathrm{Y}_{n+k}^{(h)} \right\rangle = {} & \frac{2\lambda\mu_c\alpha\theta^2(1+\alpha)(\mu_c-1)}{\mathrm{var}\left\langle \mathrm{Y}_i^{(h)}\right\rangle\beta\eta^3(\beta^2-\eta^2)}\times \\ & \left[\beta^3 A_2(h,k)-\eta^3 B_2(h,k)\right] \\ & +\frac{4\lambda\mu_c(\alpha\theta)^2 A_2(h,k)}{\eta^3}\end{aligned} \tag{3}$$

$$A_1(h)=\eta h-1+e^{-\eta h}\,;\quad B_1(h)=\beta h-1+e^{-\beta h}$$

$$A_2(h,k)=0.5\left(1-e^{-\eta h}\right)^2 e^{-\eta h(k-1)}$$

$$B_2(h,k)=0.5\left(1-e^{-\beta h}\right)^2 e^{-\beta h(k-1)}$$

where:

n = time interval counter
h = integer specifying time step interval of data (1 for 1 hour, 24 for 1 day, etc.)
Y_n^h = rainfall depth in the n-th time of interval h
$\mathbf{E}\left\langle \mathrm{Y}_n^{(h)} \right\rangle$ = mean rainfall depth record at h-hours

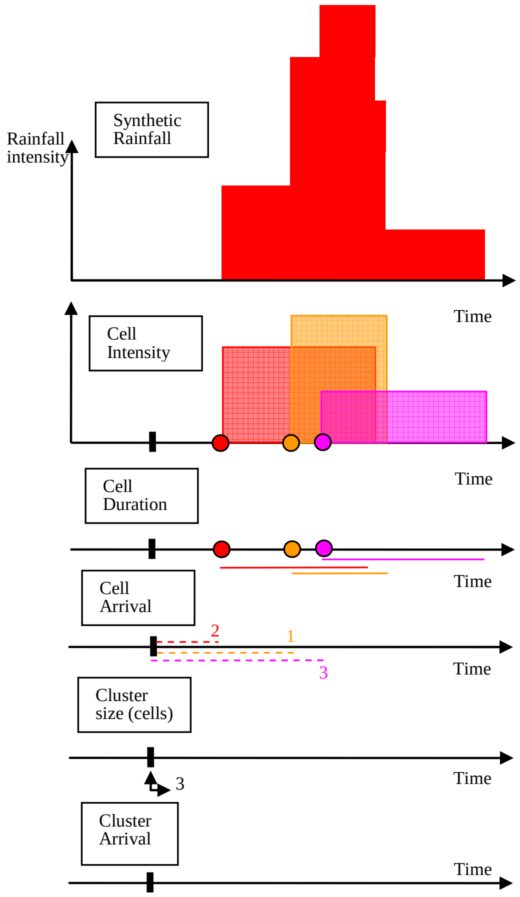

Fig. 1 Schematic drawing of the Neyman-Scott Rainfall Model.

$\mathrm{var}\left\langle \mathrm{Y}_n^{(h)} \right\rangle$ = variance of rainfall record at h-hours
$\mathrm{cor}\left\langle \mathrm{Y}_n^{(h)},\mathrm{Y}_{n+k}^{(h)} \right\rangle$ = autocorrelation of rainfall record at h-hours at lag k

Cowpertwait[4] also derived the expression for the third central moment of rainfall depth in terms of the NSM parameters. The inclusion of this term in the parameter estimation was recommended for higher consistency between synthetic rainfall maxima at multiple durations and its historical counterpart. The authors[5] have previously applied a variant of this recommended estimation procedure and found that seasonal synthetic rainfall (i.e., months June and July are designated as the Frontal Rainfall Season) are indeed more consistent due to the dependency of the third central moment on the rainfall seasonal maxima. The aggregated form of the NSM expression for the third central moment, TCM, of rainfall depth appears here as Eq. (4) where Γ() is the Gamma distribution function.

$$\mathbf{TCM}\left\langle \mathrm{Y}_n^{(h)} \right\rangle = \frac{6\lambda\mu_c \mathbf{E}\left\langle \mathrm{I}^3 \right\rangle \left(\eta h - 2 + \eta h e^{-\eta h} + 2e^{-\eta h}\right)}{\eta^4} + \frac{3\lambda \mathbf{E}\langle \mathrm{I} \rangle \mathbf{E}\left\langle \mathrm{I}^2 \right\rangle \mathbf{E}\langle \mathrm{C}(\mathrm{C}-1)\rangle F(\eta,\beta,h)}{\left[2\eta^4\beta\left(\beta^2-\eta^2\right)^2\right]} + \frac{\lambda\left(\mathbf{E}\langle \mathrm{I}\rangle\right)^3 \mathbf{E}\langle \mathrm{C}(\mathrm{C}-1)(\mathrm{C}-2)\rangle G(\eta,\beta,h)}{\left[2\eta^4\beta\left(\eta^2-\beta^2\right)(\eta-\beta)(2\beta+\eta)(2\eta+\beta)\right]} \qquad (4)$$

where:

$$\mathbf{E}\langle \mathrm{I} \rangle = \alpha\theta \, ; \quad \mathbf{E}\left\langle \mathrm{I}^2 \right\rangle = \alpha(1+\alpha)\theta^2 \, ;$$

$$\mathbf{E}\left\langle \mathrm{I}^3 \right\rangle = \theta^3 \frac{\Gamma(3+\alpha)}{\Gamma(\alpha)}$$

$$\mathbf{E}\langle \mathrm{C}(\mathrm{C}-1)\rangle = 2\mu_\mathrm{c}(\mu_\mathrm{c}-1)$$

$$\mathbf{E}\langle \mathrm{C}(\mathrm{C}-1)(\mathrm{C}-2)\rangle = 6\mu_\mathrm{c}(\mu_\mathrm{c}-1)^2$$

$$\begin{aligned} F(\eta,\beta,h) &= -2\eta^3\beta^2 e^{-\eta h} - 2\eta^3\beta^2 e^{-\beta h} + \eta^2\beta^3 e^{-2\eta h} \\ &+ 2\eta^4\beta e^{-\eta h} + 2\eta^3\beta^2 e^{-(\eta+\beta)h} - 2\eta^4\beta e^{-(\eta+\beta)h} - 8\eta^3\beta^3 h \\ &+ 11\eta^2\beta^3 - 2\eta^4\beta + 2\eta^3\beta^2 + 4\eta\beta^5 h + 4\beta\eta^5 h - 7\beta^5 \\ &- 4\eta^5 + 8\beta^5 e^{-\eta h} - \beta^5 e^{-2\eta h} - 2h\eta^3\beta^3 e^{-\eta h} - 12\eta^2\beta^3 e^{-\eta h} \\ &+ 2h\eta\beta^5 e^{-\eta h} + 4\eta^5 e^{-\beta h} \end{aligned}$$

$$\begin{aligned} G(\eta,\beta,h) &= 12\eta^5\beta e^{-\beta h} + 9\eta^4\beta^2 + 12\eta\beta^5 e^{-\eta h} + 9\eta^2\beta^4 \\ &+ 12\eta^3\beta^3 e^{-(\eta+\beta)h} - \eta^2\beta^4 e^{-2\eta h} - 9\eta^5\beta - 12\eta^3\beta^3 e^{-\beta h} \\ &- 9\beta^5\eta - 3\eta\beta^5 e^{-2\eta h} - \eta^4\beta^2 e^{-2\beta h} - 12\eta^3\beta^3 e^{-\eta h} \\ &- 3\beta\eta^5 e^{-2\beta h} - 10\beta^4\eta^3 h + 6\beta^5\eta^2 h - 10\beta^3\eta^4 h \\ &+ 4\beta^6\eta h - 8\beta^2\eta^4 e^{-\beta h} + 4h\eta^6\beta + 12\eta^3\beta^3 - 8\eta^2\beta^4 e^{-\eta h} \\ &- 6\eta^6 - 6\beta^6 - 2\eta^6 e^{-2\beta h} - 2\beta^6 e^{-2\eta h} + 8\beta^6 e^{-\eta h} \\ &+ 8\eta^6 e^{-\beta h} + 6\beta^2\eta^5 h \end{aligned}$$

(2) Model validation

Some validation of the model is necessary before proposing any intrinsic property based on its parameters. We assume that if the results based on the synthetic rainfall generated from the parameters are consistent, then both model and parameters are validated. One of the authors have shown in a previous study that the NSM is effective in the temporal modeling of rainfall from the Kyoto study region[6)] that was used in the experiments of the succeeding section. In this previous study, monthly-pooled hourly historical rainfall was used to estimate the 12 corresponding NSM parameter sets (i.e., one set per month). A hundred synthetic data sets were generated per set and compared statistically to its historical counterpart to verify the consistency of the NSM.

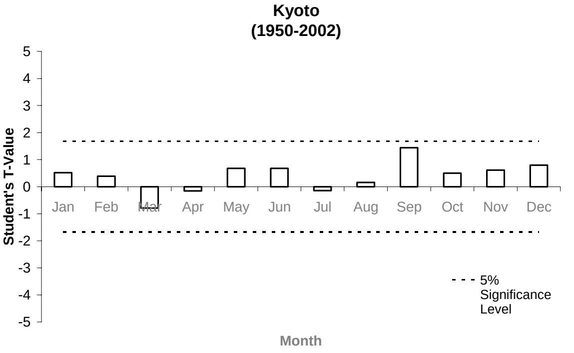

Fig. 2 Student’s T values obtained from total hourly synthetic rainfall vs. historical counterpart.

Figure 2 shows a reproduction of a verification resulting from standard Student’s T-tests on the historical and synthetic rainfall data based on the NSM. Each T-statistic in **Fig. 2** is based on the average synthetic hourly rainfall, its historical counterpart, and an appropriate estimate of standard error. The tests show that at the 5% significance level, statistical and historical rainfall means are consistent for all months. Tests for variances and autocorrelations indicate similar results. Kolomogorov-Smirnov (KS) tests on the other hand show that resulting synthetic maxima are consistent to historical counterparts at multiple durations for most months. We therefore adopt the NSM and the technique to estimate its parameters as a consistent method for temporal rainfall generation.

3. EXPERIMENTATION

(1) Kyoto historical rainfall data

The rainfall data used here was based on an hourly record obtained at Kyoto from the years 1950 to 2002. Kyoto is located in the central part of the Japanese main island of Honshu where little rainfall (as well as snowfall) is observed in the winter months such as from January to March. Frontal rainfall is observed in May, June and July. Summer rainfall in August on the other hand is often convective and torrential. In the autumn months of September and October, a large amount of rainfall is sometimes observed due to the landfall of a typhoon. We assume that these trends are perennial so that we can pool data by the month for parameter estimation.

Data is grouped into overlapping intervals such as: a) Y-A (1950 – 1970), b) Y-B (1960 – 1980), c) Y-C (1970-1990), and d) Y-D (1980-2002). Hourly rainfall is then pooled monthly per group. Sample moments from each interval is calculated, including rainfall hourly mean, hourly variance, hourly autocorrelation at lag-1, hourly third central moment, 12-hourly autocorrelation at lag-1, 24-hourly variance, and 24-hourly autocorrelation at lag-1. This combination of moments was found to yield the most consistent synthetic data generation using the NSM[5)].

(2) Parameter estimation

A basic scheme consisting of the mentioned NSM moments β, λ, μ_c, η, α, and θ is proposed based on previous studies[6, 7)]. The objective function in the parameter search is given by Eq. (5).

$$O = \sum_{m=1}^{6}\left(1-\frac{NSM_m}{HIS_m}\right)^2 \tag{5}$$

where:
m = equation index (i.e., any of equations (1), (2), (3), and (4))
NSM_m = NSM expression for rainfall moment
HIS_m = equivalent historical value of moment.

The scale parameter θ is written here in terms of the hourly mean rainfall depth as shown in Eq. (6) to reduce the computational burden of solving Eq. (5). The Levenberg-Marquardt scheme[8)] is used to carry out the optimization of the objective function.

$$\theta = \frac{\mathrm{E}\left\langle \mathrm{Y}_n^{(1)} \right\rangle \eta}{\lambda \mu_c \alpha} \tag{6}$$

where $\mathrm{E}\langle\ \rangle$ = operation to obtain expected value. The NSM parameters were estimated subject to the constraints of **Table 1**.

Table 1 Ranges in parameter estimation for NSM.

NSM Parameter	Lower limit	Upper limit
α (mm/s)	0.01	50
β (1/h)	0.05	0.99
η (1/h)	0.5	60
λ (1/h)	0.001	0.05
μ_c	1	50
θ	-	-

(3) Quantitative storm measures

Some information is available from observing the values of the NSM parameters throughout the mentioned groupings of historical data. However, we propose here several measures that describe the storm as defined in the NSM. For instance, Eq. (7) represents the approximate mean storm duration[9)] T_S which is written in terms of the NSM parameters as:

$$T_S = \frac{1}{\beta}\left(\gamma + \mathrm{Ln}\left[(\mu_c - 1)\frac{\eta}{\eta-\beta}\right]\right) \tag{7}$$

where:
γ = Euler constant, 0.5772.
Ln = natural logarithm.

Eq. (7) is a convenient estimate without resorting to complicated considerations on storm type (i.e., only monthly historical moments are required).

In addition, normalizing Eq. (1) by the duration h, yields the mean intensity of a storm:

$$I_S = \lambda \mu_c (\alpha\theta)/\eta \tag{8}$$

Similar to Eq. (7), the determination of this term depends simply on historical moments of the historical data.

The mean volume V_S of rainfall from a storm event (i.e., regardless if the storm is convective, frontal, or typhoon-based) is therefore the product of the mean duration (Eq. 7) and mean intensity (Eq. 8), or:

$$V_S = \frac{\lambda \mu_c (\alpha\theta)}{\eta\beta}\left(\gamma + \mathrm{Ln}\left[(\mu_c - 1)\frac{\eta}{\eta-\beta}\right]\right) \tag{9}$$

The equivalents of these measures based on historical rainfall cannot be readily determined. The main problem with deferring to historical data is the identification of a storm's actually start and/or end. Conversely, the NSM parameter identification procedure described previously shows no bias and arbitrariness in assessing these storm features. These are the measures we use here to determine any apparent temporal trend in historical rainfall from our study area.

4. RESULTS

(1) Overall trends

The values of the NSM model parameters were identified for each month for the four intervals Y-A (1950-1970), Y-B (1960-1980), Y-C (1970-1990), and Y-D (1980-2002). We limit our results for brevity, showing measures from several months of the year. In general, trends are difficult to detect based on plots of the parameters in time. For instance, **Fig. 2** shows the storm arrival parameter λ obtained from the historical data. Clearly, this information cannot be easily

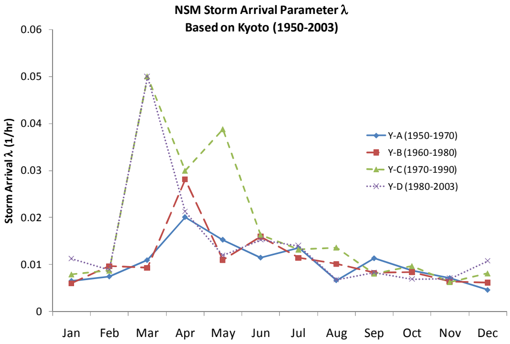

Fig. 3 Temporal trends in storm arrival parameter λ obtained from historical hourly rainfall from Kyoto, Japan.

obtained from the historical data at hand. In contrast, the parameter search used clearly shows that this parameter is directly obtained through historical moments alone. However, it is difficult to determine from this plot if a significant change occurred in this tendency throughout the entire historical record. In fact, weaker trends are observed from the other parameters and are omitted here for brevity.

(2) Monthly rain cell trends

Aside from the previous trends, other trends are easier to interpret based on monthly parameters. **Table 2** shows the NSM parameters obtained from the winter month of January. The values of λ from Y-A through Y-D indicate that more storms hit Kyoto in the current 20 year period than in the more recent 20 year periods. In addition, months similar to January with low rainfall show a fluctuating number of cells per storm (parameter μ_c) throughout the times observed.

Table 3 shows the estimated parameters for the month of June in which frontal rainfall is quite dominant. Although this month is known to experience a regular amount of rainfall in the year, the NSM parameters do not reveal any temporal trend from Y-A through Y-D. Similar to the previous findings, rain cell characteristics such as intensity and arrival do not exhibit a temporal trend throughout the historical record.

Rain cell properties differ slightly in the succeeding month of July shown in **Table 4**. The decreasing values of η indicate that the overall gap between the birth of a storm and the arrival of the rain cells has increased throughout the historical record. It also appears that the storms became more intense based on the decreasing values of average rain cell number μ_c despite the increase in rain cell duration β. The rain fronts appear to have been more severe at the end of the study period for this month.

Table 2 NSM parameter estimates and storm volume from January historical rainfall.

parameter	Y-A 1950-1970	Y-B 1960-1980	Y-C 1970-1990	Y-D 1980-2002
α (mm/s)	20.000	20.000	11.542	20.000
β (1/h)	0.316	0.292	0.171	0.369
η (1/h)	1.882	0.639	0.741	60.000
λ (1/h)	0.007	0.006	0.008	0.011
μ_c	10.036	2.222	2.170	28.284
θ	0.083	0.126	0.228	0.665

Table 3 NSM parameter estimates and storm volume from June historical rainfall.

parameter	Y-A 1950-1970	Y-B 1960-1980	Y-C 1970-1990	Y-D 1980-2002
α (mm/s)	4.649	1.605	0.123	0.074
β (1/h)	0.213	0.055	0.157	0.153
η (1/h)	59.226	0.873	2.208	1.652
λ (1/h)	0.011	0.016	0.016	0.015
μ_c	27.640	2.924	50.000	50.000
θ	13.023	3.853	7.201	9.106

Table 4 NSM parameter estimates and storm volume from July historical rainfall.

parameter	Y-A 1950-1970	Y-B 1960-1980	Y-C 1970-1990	Y-D 1980-2002
α (mm/s)	1.033	1.494	1.911	1.491
β (1/h)	0.177	0.125	0.05	0.05
η (1/h)	1.907	1.681	1.560	1.382
λ (1/h)	0.013	0.011	0.013	0.014
μ_c	5.852	4.579	3.134	3.00
θ	8.013	6.065	5.784	6.446

August and September storms show an opposite trend based on **Tables 5 and 6**. While η values show that currently rain cells arrive sooner after storm development, rain cell number μ_c also appears to have increased in the recent record. These indicate that convective storms and typhoons (August and September rainfall, respectively) have become more intense in Y-D than in Y-A.

(3) Storm trends

Figure 4 shows the effect that changes in rain cell has on overall storm intensity. There are no apparent trends based on the frontal months of June and July as well as the typhoon month of September. However, average storm rainfall intensity has steadily decreased from Y-A through Y-D in the summer month of August.

Table 5 NSM parameter estimates and storm volume from August historical rainfall.

parameter	Y-A 1950-1970	Y-B 1960-1980	Y-C 1970-1990	Y-D 1980-2002
α (mm/s)	1.222	1.110	0.808	0.290
β (1/h)	0.053	0.050	0.112	0.104
η (1/h)	1.184	1.637	2.875	2.599
λ (1/h)	0.007	0.010	0.014	0.007
μ_c	3.243	2.893	3.411	13.110
θ	9.815	10.609	14.820	17.453

Table 6 NSM parameter estimates and storm volume from September historical rainfall.

parameter	Y-A 1950-1970	Y-B 1960-1980	Y-C 1970-1990	Y-D 1980-2002
α (mm/s)	1.646	1.586	0.554	0.185
β (1/h)	0.050	0.051	0.132	0.143
η (1/h)	1.113	0.979	2.160	2.367
λ (1/h)	0.011	0.008	0.008	0.008
μ_c	2.884	3.497	19.559	50.000
θ	4.963	5.288	7.311	8.059

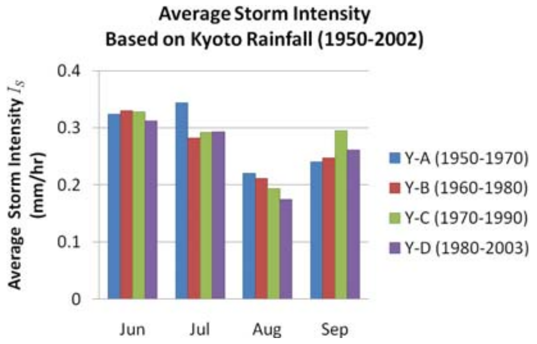

Fig. 4 Average storm intensity I_S in Kyoto, Japan.

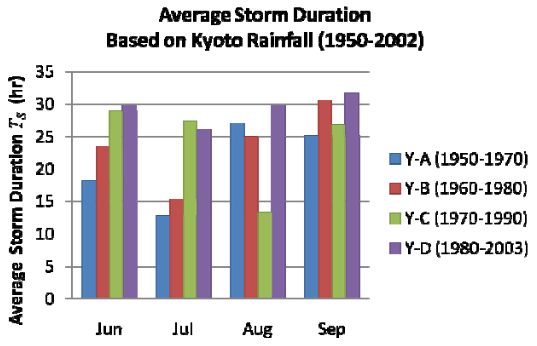

Fig. 5 Average storm duration T_S in Kyoto, Japan.

Figure 5 on the other hand shows that it is in June when storms tend to have increased in duration the most. Finally, the overall volume of rain from a storm appears to have increased in June and September, as shown in **Fig. 6**.

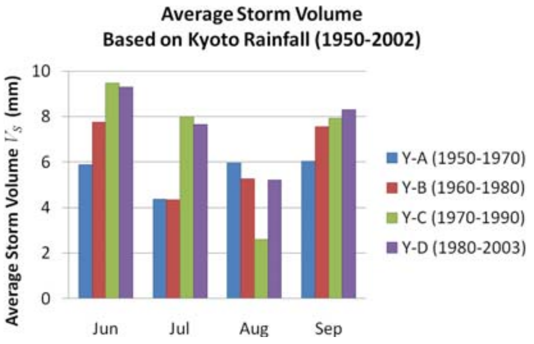

Fig. 6 Average storm volume V_S in Kyoto, Japan.

5. CONCLUSION

The Neyman-Scott clustered Poisson rectangular pulse rainfall model (NSM) was used to find a measure of temporal change in historical rainfall in Kyoto. This validated method was selected to avoid difficulties in identifying average storm characteristics such as storm duration, intensity, and volume. A 53-year (1950-2002) hourly record from Kyoto, Japan reveals trends in average overall storm volume. More severe rain fronts in June and July may have increased the rainfall throughout the mentioned period. The same was observed from the rainfall in September due to increased typhoon landfall.

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