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VALIDITY ASSESSMENT OF INTEGRATED KINEMATIC WAVE EQUATIONS FOR HILLSLOPE RAINFALL-RUNOFF MODELING

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The integrated kinematic wave model has been successfully applied to Japanese basin as an element model of distributed hydrological models in many researches. However, a recent research found that the model performance could be worse for dry and mildly sloped basins. To assess the model validity for various hydrological conditions, we conducted numerical experiments using the integrated kinematic wave model and a physically-based model combining 2D Richards equation for saturated-unsaturated flow and kinematic wave model for overland flow (2D model). The numerical experiments suggested that in case of higher rainfall intensity, thinner soil depth, steeper slope angle and wet initial condition, the simulation results of the integrated kinematic wave model showed good agreement with those of the 2D model.

Key Words : *integrated kinematic wave model, hillslope runoff, distributed hydrological modeling, numerical experiment*

1. INTRODUCTION

In hydrological modeling, the kinematic wave models have been broadly used for river flood simulations. Takasao & Shiiba[1)] proposed an integrated kinematic wave model which can handle two types of flows, matrix flow and overland flow. Their model can be applied to hillslope runoff simulation. Tachikawa et al.[2)] extended this kinematic wave model and proposed a new integrated kinematic wave model which can deal with three different flows, unsaturated, saturated, and overland flows. These integrated kinematic wave models have been successfully applied to Japanese basins as an element model of distributed hydrological models in many researches. Hunukumbra[3)] extended this model application and applied the models to several places in the world. He found a tendency that the model performances were good in wet and steep basins such as Japanese basins, while, in dry and mildly sloped basins, the integrated kinematic wave models could not give well agreed results with observation data.

Hunukumbra's finding is obviously important and critical in distributed hydrological modelings, but it is still unclear which hydrological condition effects the applicability of the integrated kinematic wave model, because only four different places were used in the study of Hunukumbra[3)]. To assess a correlation between the model validity and the each hydrological conditions, it is required to analyze the model performance in various places with many events. However, it is difficult to conduct a research using observation data because of limited available data and uncontrollable nature conditions. In this study, we used numerical experiment to assess the validity of the integrated kinematic wave model.

In this study, the results simulated by a physically-based 2D model was considered as a surrogate of observations. The 2D model consists of a 2D subsurface flow model and a 1D overland flow model. Even though 2D Richards equation cannot perfectly simulate a runoff process of real hillslope in all aspect because of many uncertainty of soil properties, structures, unknowness of hydrological processes and etc., the same model concept has been used in several previous researches for simulating hillslope runoff and it successfully reproduces the runoff discharge and the other characteristics of hillslope. For example, Hopp

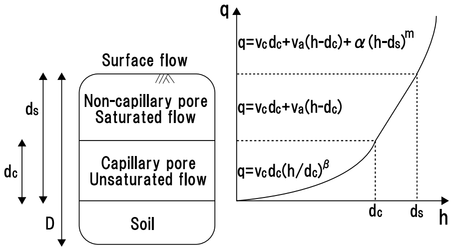

Fig.1 Concept of integrated kinematic wave model [2].

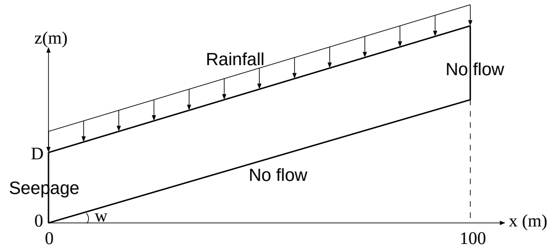

Fig.2 2D model, the subsurface flow domain.

& McDonnell[4] performed numerical experiments controlling storm size, slope angle, soil depth and bedrock permeability to study the effect for hillslope runoff using a model which is based on the same concept with the 2D model of this study. Keim et al.[5] also performed virtual experiments to investigate the process of evaporation and canopy interception using a model based on the same concept. Hence, the 2D model is considered to be useful to evaluate the effect of several hydrological conditions on runoff discharge.

Under several conditions with controlling slope angle, soil depth, total rainfall and initial condition, the simulations were conducted using the 2D model. The integrated kinematic wave model was calibrated to reproduce the results of the 2D model.

2. MODEL DESCRIPTION

(1) Integrated kinematic wave model

Takasao & Shiiba[1] proposed an integrated kinematic wave model which can handle matrix flow and overland flow in one system based on relation between water depth and discharge. Tachikawa et al.[2] modified this relation and proposed an extended model describing three types of flows. The extended model equation is as follows

$$q = \begin{cases} v_c(h/d_c)^{\beta}, & 0 \leq h \leq d_c \\ v_c d_c + v_a(h-d_c), & d_c < h \leq d_s \\ v_c d_c + v_a(h-d_c) + \alpha(h-d_s)^m, & d_s < h \end{cases} \quad (1)$$

where h is water depth, q is flow discharge, $v_c(= k_c i)$ is velocity of unsaturated flow, $v_a(= k_a i)$ is velocity of saturated flow, $\alpha(= \sqrt{i}/n)$ and β are parameters, and the shallow rectangular cross-section was assumed. To keep continuity of flow velocity, $K_c = K_a/\beta$ is assumed, where K_c and K_a are hydraulic conductivities of the unsaturated and saturated flows, respectively. Fig. 1 shows the concept of this model. There are five parameters (n, k_a, d_s, d_c, β). If d_c, the water depth corresponding to maximum unsaturated flow, is zero, this equation results in the model proposed by Takasao & Shiiba[1]. Combining Eq. 1 and the following continuity equation describes slope runoff system.

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t) \quad (2)$$

where t is time, x is horizontal dimension, r is rainfall intensity.

(2) 2D model

The physically-based 2D model consists of a 2D subsurface flow model and a 1D overland flow model. Subsurface flow is described by 2D Richards equation as follows

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left\{K\frac{\partial \psi}{\partial x}\right\} + \frac{\partial}{\partial z}\left\{K\left(\frac{\partial \psi}{\partial z} + 1\right)\right\} \quad (3)$$

where ψ is the pressure head, θ is the volumetric moisture content, K is hydraulic conductivity, x denotes the horizontal dimension, and z denotes the vertical dimension, assumed to be positive upwards. In this study, a coordinate transformation technique was used to express non-orthogonal slope as Fig. 2 but a detailed numerical scheme would not be shown in this paper due to page limitation.

The upper ($x = 100$ m) and bottom ($z - x\sin w = 0$ m) boundaries were set to be no flow. The lower boundary treated as a seepage face. Seepage face length was controlled automatically according to Neuman[6]. At the ground surface, water can enter the soil domain at the rainfall intensity (Neumann boundary) as long as ψ is negative, otherwise, it becomes Dirichlet boundary ($\psi = 0$). If the infiltration rate excesses the rainfall intensity while the surface node is Dirichlet boundary, it turns Neumann boundary condition again. This control is conducted in iterative procedure.

When the surface condition is saturated, the infiltration rate was calculated by the subsurface flow

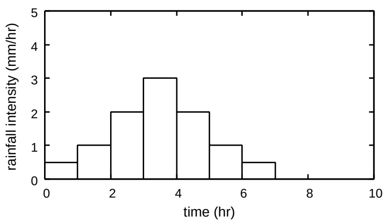

Fig.3 Rainfall intensity, total rainfall is 10 mm.

model. The amount of difference between the rainfall intensity and the calculated infiltration rate is the lateral inflow for the overland flow from soil domain and precipitation. Otherwise, when the surface condition is Neumann boundary, the infiltration rate is same as rainfall intensity and the lateral inflow for the overland flow would be zero. The overland flow equation is written as follows

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = q_l(t) \tag{4}$$

$$q = \alpha h^m \tag{5}$$

where q_l is lateral flow rate into surface flow from subsurface flow and precipitation which is calculated by the subsurface flow model. The 2D subsurface flow model and the 1D overland flow model are combined explicitly.

3. NUMERICAL EXPERIMENT

(1) Simulation condition

We considered the results simulated by the 2D model as a surrogate of observations. Under several conditions, the simulations were conducted using the 2D model and the integrated kinematic wave model was calibrated to reproduce the results of the 2D model as better as possible.

A simple slope was considered as Fig. 2. The slope length was 100m. The slope angle and soil depth were varied in simulation as Table. 1. Rainfall intensity was given as Fig. 3 when total rainfall is 10 mm. When total rainfall is 20, 40, 70, and 100 mm, the rainfall intensity was multiplied by 2, 4, 7 and 10, respectively. Two types of initial conditions were considered. A wet initial condition was simulated as soil moisture contents and the corresponding pressure head field after three days drainage from a saturated soil domain without rainfall. A dry initial condition was obtained after a week drainage.

Table.1 Simulation condition

slope angle	5, 20, 35 degree
soil depth	0.25, 0.5, 1.0, 2.0 m
total rainfall	10, 20, 40, 70, 100 mm
initial condition	wet, dry

(2) Model parameter

The equation of Van Genuchten[9] for the soil water retention curve and that of Mualem[10] for the unsaturated hydraulic conductivity function were used for the subsurface flow model. The soil water retention curve is given by

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left\{ \frac{1}{1 + (\alpha|\psi|)^n} \right\}^{1-1/n} \tag{6}$$

where S_e is the effective saturation; θ_r and θ_s are the residual and saturated water contents, respectively; and α and n are Van Genuchten parameters, whose values depend upon the soil properties. Based on the model of Mualem[10], the unsaturated hydraulic conductivity function is given by

$$K = K_s S_e^{1/2} \left\{ 1 - (1 - S_e^{n/(n-1)})^{1-1/n} \right\}^2 \tag{7}$$

where K_s is the saturated hydraulic conductivity. The parameter values were set as $\theta_s = 0.475$, $\theta_r = 0.28$, $K_s = 2.5$ m/s, $\alpha = 4$ m^{-1}, $n = 2$ by referring to Hopp & McDonnell [4].

As the model of Tachikawa et al.[2] includes that of Takasao & Shiiba[1], the former one was used as the integrated kinematic wave model in this study. The five parameters (n, k_a, d_s, d_c, β) of the integrated kinematic wave model were calibrated for each simulations of the 2D model by using the SCE algorithm[7]. The Nash Sutcliffe coefficient[8] was used as the objective function of parameter optimization.

4. RESULT AND DISCUSSION

(1) Runoff discharge

Fig. 4 describes four examples of runoff discharge simulated by the 2D model and the integrated kinematic wave model. The left column of the figure shows the results of the 2D model and the right column shows those of the integrated kinematic wave model. When soil depth is 0.25 m and 1 m, the runoff discharges simulated by the two models are well matched. On the other hand, the discharge patterns are comparatively not agreed each other with 2m soil depth. The

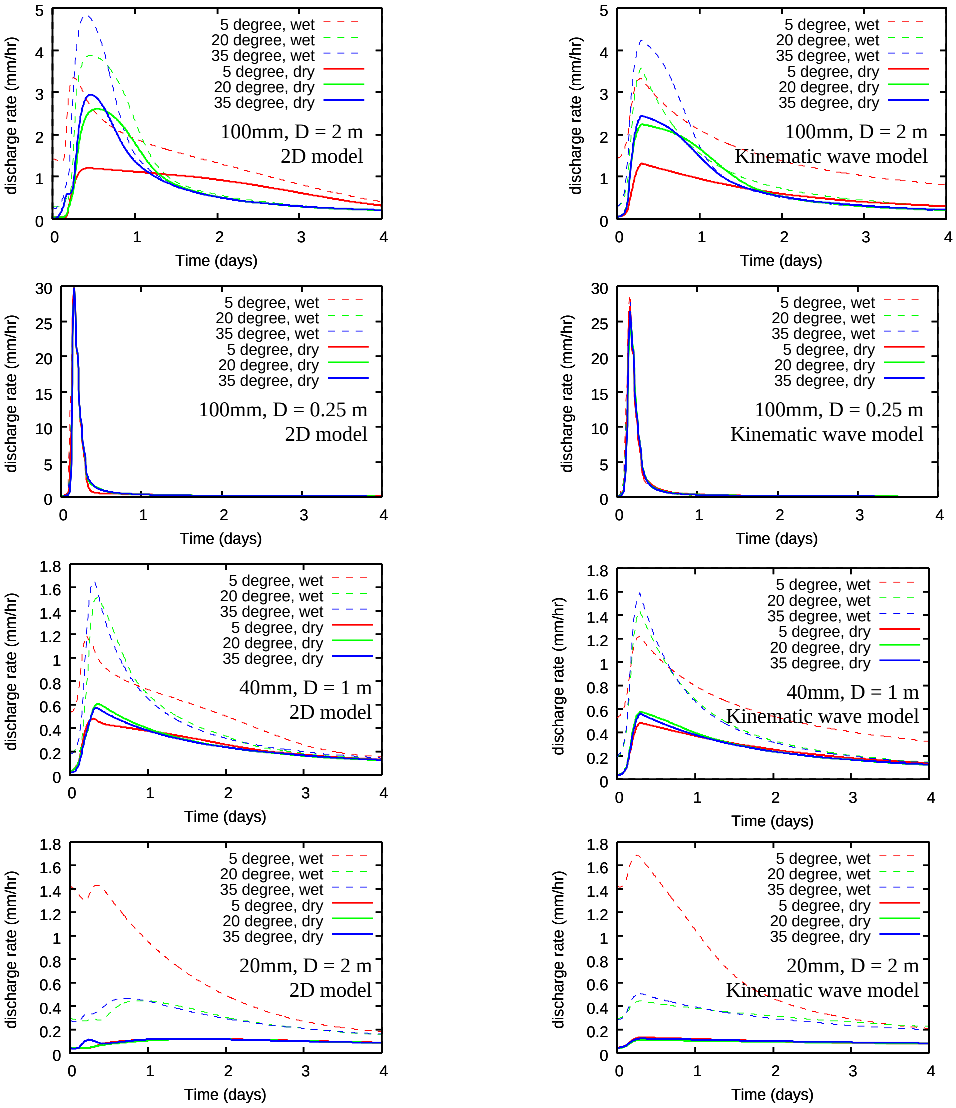

Fig.4 Hydrographs simulated by 2D model and integrated kinematic wave model.

peak time of the integrated kinematic wave model appeared earlier than that of the 2D model with 2m soil depth. This difference is supposed to be arised from vertical infiltration which cannot be treated in the integrated kinematic wave model. However, the overall runoff pattern was well reproduced entirely. The slope of 5 degree results in earlier peak flow than the others sometime. This is caused by the difference of initial condition which of 5 degree slope is wetter that the others. As previously mentioned in **3.(1)**, the initial conditions were made by simulating three days drainage from a saturated soil domain without rainfall. Hence, the initial conditions of 5 degree slope is generally wetter than the other two conditions.

(2) Nash Sutcliffe coefficient

Fig. 5 shows the Nash Sutcliffe coefficients with different soil depths. If the integrated kinematic wave model perfectly reproduces a runoff discharge pattern simulated by the 2D model, the corresponding Nash Sutcliffe coefficient becomes 1. Obviously, Nash Sutcliffe coefficients become worse as soil depth becomes thicker. It could be arised from the effect of vertical infiltration. As soil depth is thicker, vertical infiltration gives larger effect to runoff process. Because the integrated kinematic wave model does not have a mechanism describing vertical infiltration in soil domain, it fails to reproduce the runoff discharge calculated by the 2D model which can express effect of vertical infiltration in thick soil condition. Furthermore, Nash Sutcliffe coefficients tend to be higher with wet initial

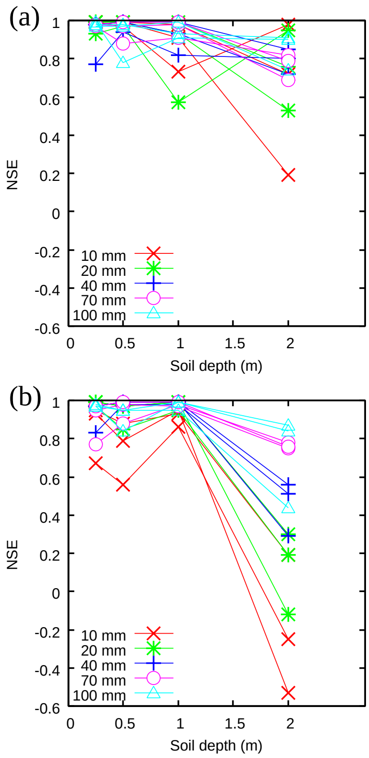

Fig.5 Nash Sutcliffe coefficient with different soil depths: (a) wet, (b) dry initial condition.

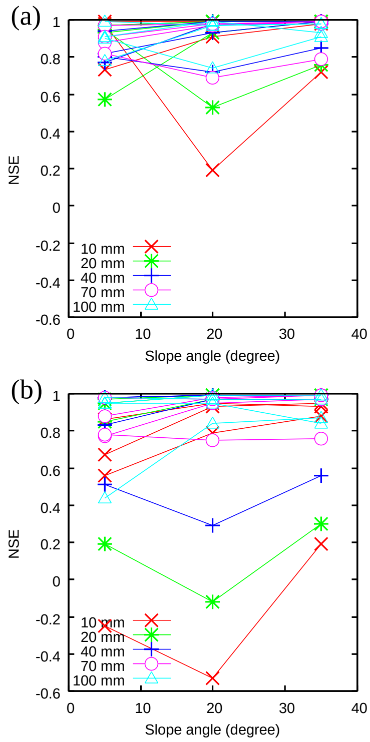

Fig.6 Nash Sutcliffe coefficient with different slope angles: (a) wet, (b) dry initial condition.

conditions and vice versa with dry initial condition. This reason also might be two dimensional flow effect. As the initial condition is wetter, the soil domain is easily and quickly saturated and the vertical infiltration gives relatively small effect to runoff process.

Fig. 6 shows the Nash Sutcliffe coefficients with different slope angels. It seems like that there is no apparent correlation between slope angle and validity of the integrated kinematic wave model. However, except considerably wrong results (Nash Sutcliffe coefficient less than 0.7), reproducibility tends to be better with larger slope angle. This might be arised from that flow condition of the lower part does not affect that of upper part in the kinematic wave model because it assumes supercritical flow.

These two characteristics shown in Fig. 5 and 6 about validity of the integrated kinematic wave model are well agreed with the research of Hunukumbra[3]. He found that the performance of the integrated kinematic wave model was good in wet and steep basins while it becomes worse in dry and mildly sloped basins.

Another point is that the validity is better with high rainfall intensity in Fig. 5 and 6. The reason is supposed to be same as that of the initial condition effect. In high rainfall intensity, vertical infiltration gives less effect to overall runoff process.

(3) Discharge components

As previously mentioned, if $d_c = 0$ in the model proposed by Tachikawa et al.[2], the model results in that proposed by Takasao & Shiiba[1].

Fig. 7 shows separate components of discharges of three models with 100 mm total rainfall, $D = 0.25$ m, 5 degree sloep angle and wet initial condition. It can be found that the 2D model and the model proposed by Tachikawa[2] give different discharge components even though the overall discharges are well agreed each other. The two integrated kinematic wave models gave different results and the hydrograph simulated by Takasao & Shiiba's model[1] is closer to the 2D model's result. However, as Takasao & Shiiba's model[1] can be considered as a variant of Tachikawa's model[2], the result simulated by Takasao & Shiiba's model[1] can be considered as the result of Tachikawa's model[2]. This is a typical parameter identification problem. It means that another information is required to reproduce discharge components using the integrated kinematic wave model, e.g. specific param-

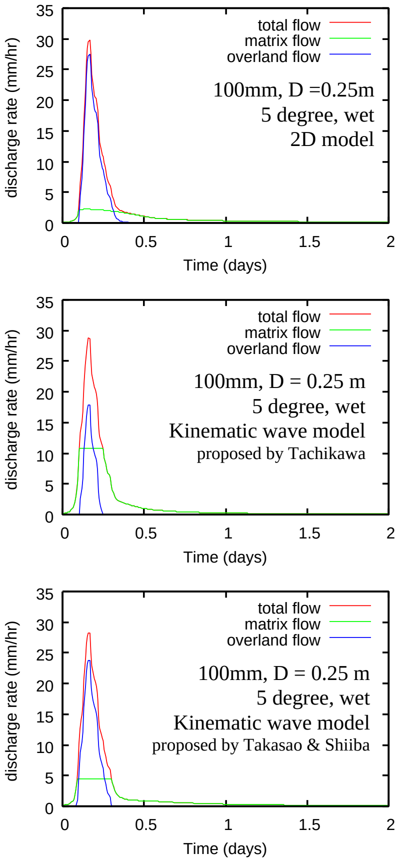

Fig.7 Matrix and overland flows simulated by the 2D model and the integrated kinematic wave model proposed by Tachikawa et al.[2] and Takasao & Shiiba[1] (100 mm, $D = 0.25$ m, 5 degree, wet).

eter range or discharge rate of specific component.

5. CONCLUSION

To assess the validity of the integrated kinematic wave model, we conducted numerical experiments using the physically-based 2D model and the integrated kinematic wave model. The results simulated by the 2D model were assumed to be a surrogate of observations and the kinematic wave model was calibrated to reproduce the result of the 2D model. It was supposed that rainfall intensity, soil depth, slope angle, and initial condition could effect to the validity of the integrated kinematic wave model and the numerical experiments were conducted with changing these factors. According to the results of the numerical experiments, in condition of higher rainfall intensity, thinner soil depth, steeper slope angle, and wet initial condition, the integrated kinematic wave model could give good agreement with the 2D model. This analysis corresponds with the study of Hunukumbura[3]. Including vertical infiltration mechanism, the integrated kinematic wave model may give better result for dry and mildly sloped basins.

Another important finding is that if the integrated kinematic wave model perfectly reproduces the overall discharge from hillslope, it does not guarantee the good agreement with respect to discharge components. To obtain reliable result of separated discharge components using the integrated kinematic wave model, the other information which limits the range of model parameter should be needed in terms of hydrological characteristics of the study basins.

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