水文学基礎 Fundamentals of Hydrology (流出 Runoff) Homework Answer

The kinematic wave equations for overlandflow on a rectangular slope are derived as

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r \tag{1}$$

$$q = \alpha h^m \tag{2}$$

Suppose that the flow depth h is 0 anywhere on a rectangular slope at time $t = 0$ and rainfall intensity r takes a constant value r_0 from $t = 0$ to $t = T$. At $t = T$ rainfall stops and for $t > T$ the rainfall intensity r is 0. The slope length is L. T is supposed to be less than $\{L/(\alpha r_0^{m-1})\}^{1/m} = t_c$. This condition means the rainfall stops before the characteristic curve starting at the upper end of the slope reaches to the lower end of slope, because T is smaller than the time of concentration. In this case, Calcurate the time that the characteristic curve starting at the top of slope at $t = 0$ reaches the slope end.

矩形平面上のキネマティックウェーブモデルを考える。時刻 $t = 0$ で水深 h は斜面のどこでも 0 であり、降雨強度 r は、時刻 $t = 0$ から $t = T$ まで一定値 r_0 をとり、その後、降り止んで $r = 0$ になるとする。斜面長は L とする。ここで降雨期間 T は、$T < \{L/(\alpha r_0^{m-1})\}^{1/m}$ を満たしているとする。一定の降雨が継続するとして得られる伝播時間 $t_c = \{L/(\alpha r_0^{m-1})\}^{1/m}$ よりも短い時間で降雨が終了することに注意すること。このとき、$t = 0$ に斜面上端を発した特性曲線法が斜面下端に到達する時間を求めよ。

(Answer)

The characteristic curve starting at $x = 0$ reaches

$$x(T) = \alpha {r_0}^{m-1} T^m \tag{3}$$

at $t = T$. At this time, the flow depth is given by

$$h(T) = r_0 T \tag{4}$$

For $t \geq T$, rainfall intensity is zero. Thus the characteristic curve for $t \geq T$ is obtained as

$$\int_{x(T)}^{x(t)} d\xi = \int_T^t \alpha m \{h(T)\}^{m-1} d\tau = \alpha m \{h(T)\}^{m-1} \int_T^t d\tau$$

$$\therefore\ x(t) = x(T) + \alpha m (r_0 T)^{m-1}(t - T) = \alpha {r_0}^{m-1} T^m + \alpha m (r_0 T)^{m-1}(t - T) \tag{5}$$

The time t_C when the characteristic curve reaches at the end of slope is given by setting $x(t_C) = L$. Thus,

$$L = \alpha {r_0}^{m-1} T^m + \alpha m (r_0 T)^{m-1}(t_C - T) \tag{6}$$

t_C is given by

$$t_C = \frac{L - \alpha r_0^{m-1} T^m}{\alpha m (r_0 T)^{m-1}} + T \tag{7}$$