Lumping slope runoff model using digital elevation model and steady state assumption

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1 Summary

A lumped runoff model is developed by spatially integrating kinematic wave equation in order to reduce the computation burden of water flow routing in the slope area. To spatially integrate kinematic wave equation, the digital topographic information and steady state assumption are used. The hydrographs calculated by this lumped model are compared with those by the distributed kinematic wave model. This comparison shows that hydrographs simulated by the lumped model are similar to those by the distributed model, while the computation time of this model is much shorter than that of the distributed model. The hydrographs calculated by the lumped model are also compared with observed data.

2 Introduction

A number of digital elevation models (DEMs) have been developed using digital topographic information, and applications of rainfall-runoff models to DEMs have been also carried out. The simplest method to apply a runoff model to a DEM is to route water flow on a grid-based DEM from a grid point to one of its eight neighbors in the direction with the steepest downward slope. Since spatial resolution of digital topographic information has become higher in recent years, it becomes possible to more precisely model basin topography. On the other hand, the applications of such type of water flow routing model to high spatial resolution DEMs require much computation time and memory.

In this paper we develop a lumped kinematic wave runoff model by spatially integrating kinematic wave equation in order to reduce computation burden of water flow routing in the slope area. To spatially integrate kinematic wave equation, digital topographic information and steady state assumption are used. This lumped model is on the physical basis and its parameter value is calculated using topographic variables obtained from DEM of the study area.

At first we present the lumping method of kinematic wave equation using digital topographic information and steady state assumption. We then show applications of this lumped model and compare its results with those of the "distributed" kinematic wave model. Also we compare outflow hydrographs simulated by the lumped model with observed data.

3 Lumping kinematic wave model

3.1 DEM used

In this paper we use a grid-based DEM to develop the lumped kinematic wave runoff model. Figure 1 shows a schematic representation of the grid-based DEM which we use. In this figure two adjacent grid points are connected by a rectangular plane. We call this rectangular plane "a slope unit". Basin topography is represented by a set of slope units. For each slope unit, its area, length and gradient can be easily calculated. Therefore we can easily obtain the upslope contributing area at a given location which will be a key point to lumping kinematic wave model.

3.2 Lumping method

Kinematic wave equation is expressed as $A(x) = KQ(x)^P$, in which $A(x)$ is the flow cross-sectional area, $Q(x)$ is the flow discharge, and K and P are coefficients. In order to spatially integrate kinematic wave equation, we introduce two following assumptions:

assumption 1: it is assumed that the rainfall-runoff system in the study area is in the steady state by spatially uniform rainfall input. For such assumption, $Q(x)$ is expressed as $Q(x) = r_0 M(x)$, in which r_0 is the rainfall intensity, $M(x)$ is the upslope contributing area at some location. We call this assumption "steady state assumption".

assumption 2: it is assumed that the coefficient in kinematic wave equation, P, is spatially uniform.

Note that the assumption 2 means runoff processes in the study area are considered to be spatially uniform in this lumping method.

Suppose that the study area is represented by N slope units. Let us consider the ith slope unit ($i = 1, 2, ..., N$). Let L_i be the length, B_i the width, M_i the area, G_i the gradient, and K_i the kinematic coefficient K, of the ith slope unit. K_i is expressed as:

$$K_i = \begin{cases} \left\{ n_i B_i^{\frac{1}{P}-1} / \sqrt{\sin(G_i)} \right\}^P & \text{(for surface flow)} \\ \gamma_i / (k_i \sin(G_i)) & \text{(for subsurface flow)} \end{cases} \tag{1}$$

where n_i is the Manning coefficient, k_i is the hydraulic conductivity, and γ_i is the effective porosity. Also let U_i be the upslope contributing area of the ith slope unit. If the ith slope unit is located on the upper boundary of the study area, $U_i = 0$.

Let $A_i(x)$ be the flow cross-sectional area and $Q_i(x)$ the flow discharge at the distance x from the upper end of the ith slope unit. Since a slope unit is a rectangular plane, the upslope contributing area at the distance x from its upper end of the ith slope unit, $M_i(x)$, is expressed as:

$$M_i(x) = M_i x / L_i + U_i \tag{2}$$

Then from the assumption 1, we have

$$Q_i(x) = r_0 (M_i x / L_i + U_i) \tag{3}$$

and from kinematic wave equation, we have

$$A_i(x) = K_i r_0^P (M_i x / L_i + U_i)^P \tag{4}$$

Spatially integrating $A_i(x)$, we have the storage in the ith slope unit, s_i

$$s_i = \int_0^{L_i} A_i(x) dx = \frac{r_0^P}{P+1} \frac{K_i L_i}{M_i} \left\{ (M_i + U_i)^{P+1} - (U_i)^{P+1} \right\} \tag{5}$$

Summing up s_i, we have the storage in the study area, S

$$S = \sum_{i=1}^{N} s_i = \frac{r_0^P}{P+1} \sum_{i=1}^{N} \frac{K_i L_i}{M_i} \left\{ (M_i + U_i)^{P+1} - (U_i)^{P+1} \right\} = E r_0^P \tag{6}$$

where

$$E = \frac{1}{P+1} \sum_{i=1}^{N} \frac{K_i L_i}{M_i} \left\{ (M_i + U_i)^{P+1} - (U_i)^{P+1} \right\} \tag{7}$$

Let O be the outflow discharge from the study area. Under the steady state assumption,

$$r_0 = O / \sum_{i=1}^{N} M_i \tag{8}$$

Substituting Equation (8) into (6), we obtain

$$S = E \left(\frac{O}{\sum_{i=1}^{N} M_i} \right)^P = F O^P \tag{9}$$

where $F = E / (\sum_{i=1}^{N} M_i)^P$.

Equation (9) was derived under the steady state assumption. However we use this equation to calculate outflow discharge for the unsteady state, that is, time variant rainfall case. In this case we have the relationship between storage and outflow as follows:

$$S(t) = F O(t)^P \tag{10}$$

Continuity equation is given by

$$\frac{dS}{dt} = Q_L(t) - O(t) = Q_L(t) - \left(\frac{S(t)}{F} \right)^{\frac{1}{P}} \tag{11}$$

where $Q_L(t)$ is rainfall intensity. Since Equation (11) is an ordinary differential equation for $S(t)$, we numerically solve this equation by using Runge-Kutta method. We call the model derived here "the lumped kinematic slope model".

Table 1: the subcatchment areas and the number of slope units

	area(km^2)	number of slope units
subcatchment A	19.35	7916
subcatchment B	27.88	11317
subcatchment C	32.76	13349

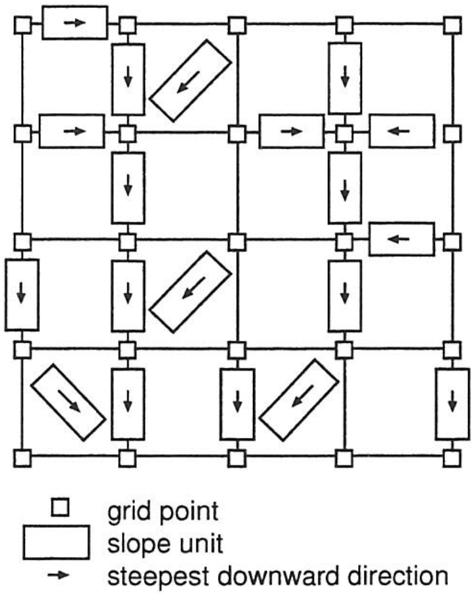

Figure 1: DEM used in this study

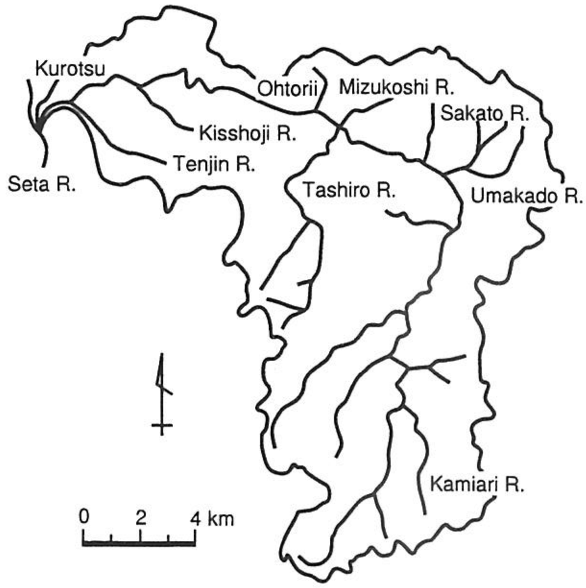

Figure 2: The map of Daido River catchment

4 Applications

4.1 Study area

The study area is the Daido River catchment located near Biwa Lake which is the largest lake in Japan. The area of the catchment is 189.5 km^2. Hill slope gradients are relatively steep. Figure 2 shows the map of the study area.

4.2 Comparing outflows calculated by the lumped kinematic slope model with those by the distributed kinematic wave model

To validate the performance of the lumped kinematic slope model, we apply it to the study area and compare its results with those by the distributed kinematic wave model under same rainfall / model parameter conditions. The study areas for this application are three subcatchments in the Daido River catchment. Table 1 summerizes the subcatchment areas and the number of slope units which compose each subcatchment.

4.2.1 Distributed kinematic wave model

In this paper we call the usual form of kinematic wave model "the distributed kinematic wave model". We solve the distributed kinematic wave equation by using a finite difference method described in the following.

Kinematic wave equation is given by

$$A(x,t) = KQ(x,t)^P \tag{12}$$

where $A(x,t)$ is the flow cross-sectional area, $Q(x,t)$ is the flow discharge, and K and P are coefficients. $A(x,t)$ and $Q(x,t)$ are abbreviated to A and Q, respectively in the following. Continuity equation is expressed as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q(t) \tag{13}$$

where $q(t)$ is the lateral inflow discharge per unit length. Equation (13) is rewritten as

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = cq(t) \tag{14}$$

where c is the kinematic wave velocity ($= dQ/dA$).

Discretizing Equation (14) by the scheme developed by Beven[1], we have

$$\frac{Q_{i,t+1} - Q_{i,t}}{\Delta t} + \theta c_{i-1/2,t+1}\left(\frac{Q_{i,t+1} - Q_{i-1,t+1}}{\Delta x} - q\right) + (1-\theta)c_{i-1/2,t}\left(\frac{Q_{i,t} - Q_{i-1,t}}{\Delta x} - q\right) = 0 \tag{15}$$

where $c_{i-1/2} = 0.5(c_i + c_{i+1})$ and θ is the time weighting parameter. In this equation the unknown variable is $Q_{i,t+1}$. Collecting unknown terms to the left hand side of the equation we obtain

$$\left(1 + \theta c_{i-1/2,t+1}\frac{\Delta t}{\Delta x}\right) Q_{i,t+1} = Q_{i,t} + \theta c_{i-1/2,t+1}\left(\frac{\Delta t}{\Delta x}Q_{i-1,t+1} + \Delta t q\right) - (1-\theta)c_{i-1/2,t}\Delta t\left(\frac{Q_{i,t} - Q_{i-1,t}}{\Delta x} - q\right) \tag{16}$$

Equation (16) is a nonlinear equation of $Q_{i,t+1}$ because $c_{i-1/2,t+1}$ is expressed by $Q_{i,t+1}$. To solve this equation, we use the following iterative procedures:

step 1: Let $\hat{Q}_{i,t+1}$ be the estimate of $Q_{i,t+1}$. Give the initial value to $\hat{Q}_{i,t+1}$.

step 2: Calculate $c_{i-1/2,t+1}$ substituting $\hat{Q}_{i,t+1}$ into $Q_{i,t+1}$.

step 3: Calculate $Q_{i,t+1}$ taking $\left(1 + \theta c_{i-1/2,t+1}\Delta t/\Delta x\right)$ to be constant.

step 4: If $|Q_{i,t+1} - \hat{Q}_{i,t+1}| \leq \epsilon$ (tolerance level), go to calculation of $Q_{i+1,t+1}$. Otherwise, consider $Q_{i,t+1}$ obtained in step 3 as the new estimate value and go back to step 2.

Applying the above method to all the slope units in the study area from upstream to downstream, we can calculate the outflow discharge from the study area.

4.2.2 Surface flow simulations

In order to compare the lumped kinematic slope model with the distributed kinematic wave model, runoff simulations are carried out on the assumption that only surface flow occurs in the study area. We call this type of simulation "a surface flow simulation". Initial values and model parameters used are as follows: initial storage in the study area is 0 m^3, initial outflow from the study area is 0 m^3/s, Manning coefficient n is 0.3 m-s, and kinematic coefficient P is 0.6. n and P are uniform in space. Figure 3 shows the results of simulations. From this figure it can be said that the hydrographs simulated by the two models are very similar in all study areas. It implies that the steady state assumption used in the lumped model is appropriate for the surface flow simulation. The computation time by the lumped model was much shorter than that by the distributed model.

4.2.3 Subsurface flow simulations

Subsurface flow simulations are also carried out. Rainfall condition and initial state of the models are the same as those of the surface flow simulations. The parameter values used are as follows: hydraulic conductivity k is 0.2 m/s, effective porosity γ is 0.1, and kinematic coefficient P is 1.0. These parameter values are spatially uniform.

The results of simulations are shown in Figure 4. Figure 4 shows that differences between hydrographs simulated by the two models are slightly larger than those of surface flow simulations. Since kinematic wave velocity of subsurface flow is generally smaller than that of surface flow, subsurface flow requires longer time to become steady state than surface flow. In other words, subsurface flow has stronger unsteadiness as compared with surface flow. Thus it can be said that subsurface flow simulations are inconsistent with the steady state assumption used in the derivation of the lumped kinematic slope model, and this inconsistency may cause relatively large differences between the results of the two models. However since the differences are not so large, the steady state assumption seems to be effective. The computation time by the lumped model was shorter than that by the distributed model in the same way as the surface flow simulations.

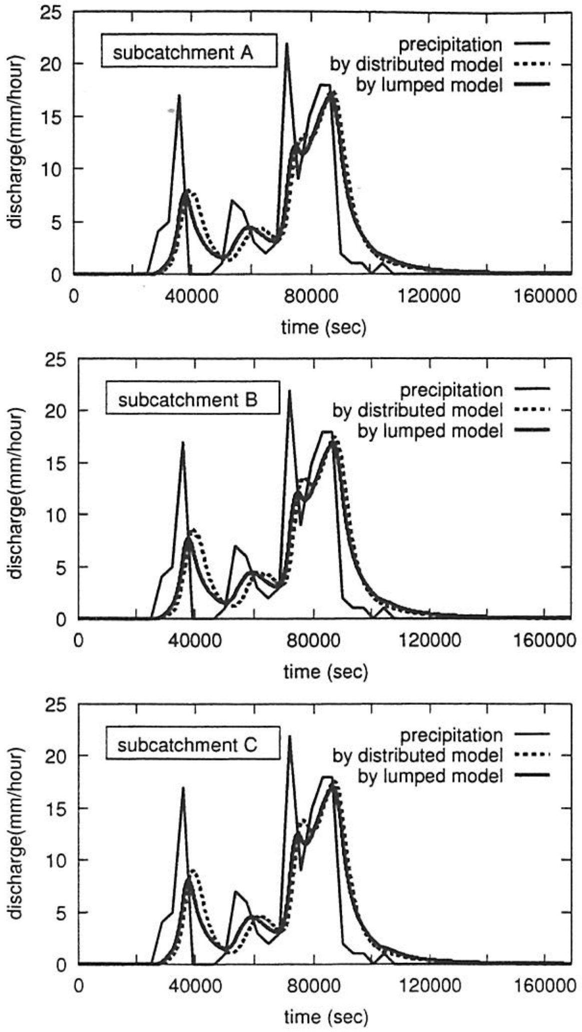

Figure 3: Surface flow simulations

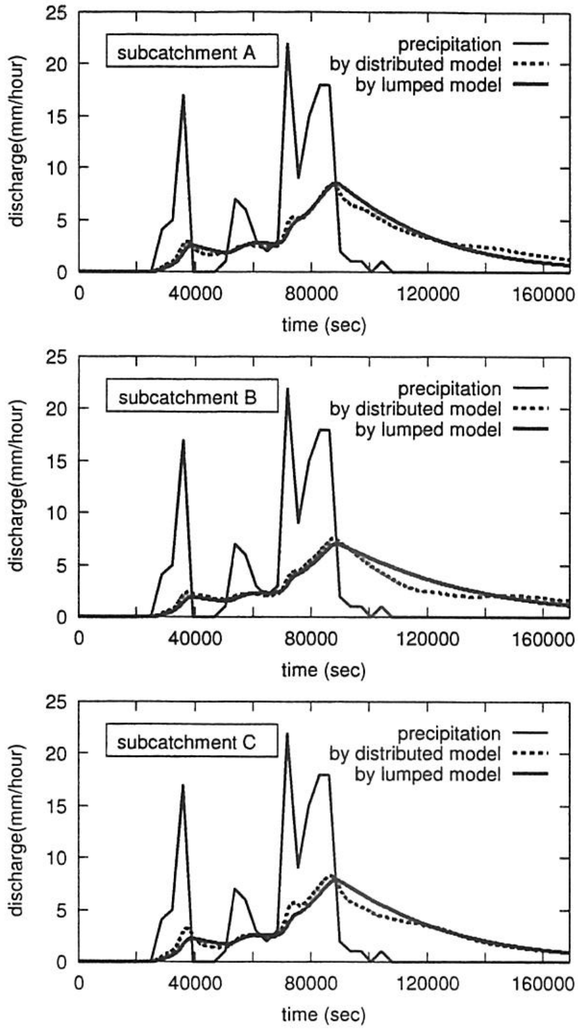

Figure 4: Subsurface flow simulations

4.3 Comparing discharges calculated by the lumped model with observed data

We compare the discharges calculated by the lumped kinematic slope model with observed data. The study area of this application is the whole of Daido River catchment. Stream network flow is routed by the lumped kinematic stream network model developed by Shiiba et al.[2]. Two rainfall data (case 1: 1 - 2 August 1982, case 2: 19 - 20 September 1990) are used. For each case, (a) surface flow simulation and (b) subsurface flow simulation are carried out and the results are compared with observed data.

Figures 5 and 6 show the results of case 1 and case 2, respectively. The results of surface flow simulation are shown in Figures 5(a) and 6(a), which show that the calculated discharges are quite different from observed data for both cases. This error may be caused by the assumption that only surface flow occurs in the study area in this simulation. Since calculated velocity of surface flow is much faster than the real one, the model calculates outflow hydrographs with much faster response and less attenuation of the peaks than observed data.

The results of subsurface flow simulation are shown in Figures 5(b) and 6(b). Figure 5(b) shows that the calculated discharge for case 1 is less than the observed data at the peak. This seems to be caused by that only subsurface flow is considered and surface flow is not considered in this simulation. Since case 1 has much precipitation, at some location in the study area surface runoff may occur, which may give rise to the high peak of the observed hydrograph. Figure 6(b) shows relatively good agreement between the calculated hydrograph and the observed data. Since case 2 has less precipitation than case 1, subsurface flow may be a dominant runoff process in the study area. It seems to cause the relatively good result for the subsurface flow simulation in case 2.

The results of these comparisons described above imply that the surface flow simulation is not realistic

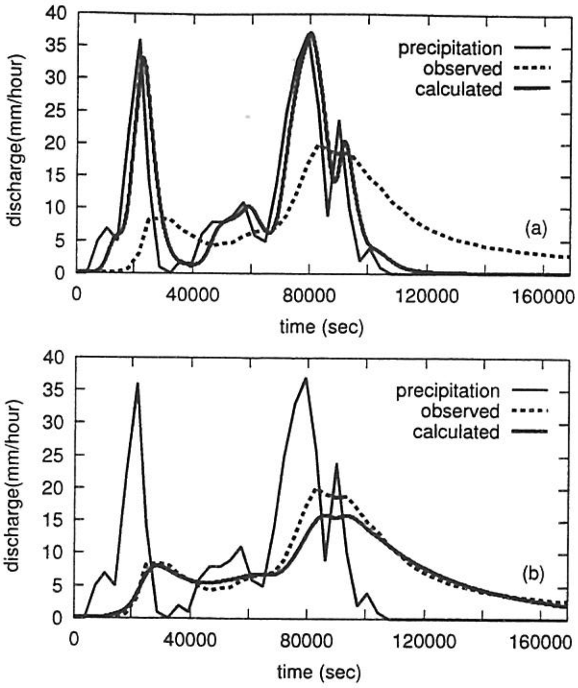

Figure 5: Simulations for case 1

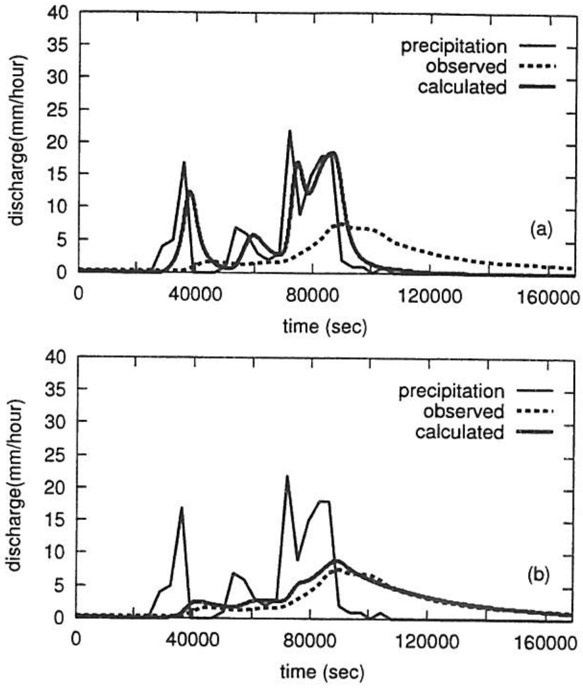

Figure 6: Simulations for case 2

and the subsurface flow simulation is also insufficient for reproducing the observed data. Thus the current lumped kinematic slope model has the limitation that it can not consider the spatial variablity of runoff processes. The method should be developed which makes it possible to integrate kinematic wave equation considering the spatial variablity of runoff processes.

5 Conclusions

We have presented a lumped kinematic slope model for slope area by spatially integrating kinematic wave equation using the digital elevation model and the steady state assumption. We compared the hydrographs calculated by the lumped kinematic slope model with those by the distributed kinematic wave model, in order to validate the performance of the lumped model. This comparison showed that hydrographs simulated by the lumped model was similar to those by the distributed model, while the computation time of the former was much shorter than that of the latter. Also we compared the hydrographs calculated by the lumped model with observed data. The lumped model did not give good results for this comparison. The errors in this comparison can be interpreted as resulting from the model assumption that runoff processes are uniform in space. It is concluded that the method should be developed which makes it possible to integrate kinematic wave equation considering the spatial variablity of runoff processes.

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[2] Shiiba, M., Y. Ichikawa, S. Ikebuchi, Y. Tachikawa, and T. Takasao (1996) : Scale-up of a runoff model using GIS and object-oriented hydrological modeling system, Application of Geographic Information Systems in Hydrology and Water Resources Management, IAHS publication, no. 235, pp. 63-70.

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Authors of these abstracts were then asked to submit their full length papers by 1 February 1999. These papers were sent to members of the National and International Scientific Committee and Australian experts in the field for review. A total of 180 papers were accepted for oral presentations and 42 as poster papers. Authors were asked to revise their papers in accordance with the reviewers' comments and send the revised papers back by 1 May 1999.

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The final proceedings were edited by Dr Walter Boughton.

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