Pioneering Studies on Kinematic Wave Models

Translated from Transactions of JSCE
the paper entitled
"FUNDAMENTAL RESEARCHES ON THE UNIT HYDROGRAPH METHOD AND ITS APPLICATION"

October, 1996

Edited by
KIYOSHI HOSHI

Why do we translate .... ?

This paper was published in March 1959 as the first research works in the world on the kinematic wave model applied to the rainfall-runoff process in mountainous catchments. A significant contribution was made by the authors who clarified the concept of "variable source area of overland flow " in the paper and pointed out importance of its role in runoff mechanisms. It was not until more than twenty years that systematic and scientific approaches were introduced in modern hydrology. Unfortunately an original paper was written in Japanese and hence their excellent study has not been recognized throughout the world for a long time. It is now worthwhile to translate it into English in honor of the authors. The following are the members of a working group who have contributed their long hours to translation processes:

With our sincere respect to the authors.

翻訳を行うに際して

本論文は１９５９年に土木学会論文集に掲載されたものであり、山地流域での流出解析にKinematic Wave法の適用を世界で最初に提案したものであります。本論文では表面流出の変動流出寄与域（variable source area）の概念がいち早く導入され、流出機構における その役割の重要性も明確に指摘されています。これらの研究は、その後の組織的・科学的研究に先駆けてほぼ20年以前に行われたものであります。残念ながら、原論文は日本語で書かれていたため、長年、その内容は世界に紹介されずに今日に至っております。そこで我々は、著者らの卓越した業績と今でも斬新な理論展開をたたえるために、ここに、以下のメンバーで論文の英文化を行ったものであります。

石原藤次郎先生、高棹琢馬先生に尊敬の念をもって英文化を実施させていただきました。

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A translated paper in English

TRANSACTIONS
OF THE
JAPAN SOCIETY OF CIVIL ENGINEERS

NO.60, EXTRA PAPER (3-3)

FUNDAMENTAL RESEARCHES ON THE UNIT HYDROGRAPH METHOD AND ITS APPLICATION

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March 1959
TOKYO JAPAN

JAPAN SOCIETY OF CIVIL ENGINEERS

FUNDAMENTAL RESEARCHES ON THE UNIT HYDROGRAPH METHOD AND ITS APPLICATION

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Synopsis: The basic principle of the unit hydrograph method assumes that there exists a linear relationship between the discharge $Q(t)$ and the rainfall $r(t)$ expressed as

$$Q(t)=\int_{-\infty}^{t}X(\tau)r(t-\tau)d\tau$$

where $X(t)$ is the unit hydrograph given as a kernel of the linear integral equation.

Since this expression is obviously based on empirical relations, the hydraulic significance of unit hydrograph method will not be clarified, unless a dynamic basis of the above expression is verified in the light of fluid mechanics.

Under these considerations, the present paper deals theoretically with the following two themes from a standpoint of hydraulics:

(i) Hydraulic significance of the fundamental principle of unit hydrograph method.

(ii) The most preferable elements of the unit rainfall and the unit hydrograph, and estimation of errors due to the application of unit hydrograph method.

It is expected that the results derived in the present paper will bring effective procedures to solve important practical problems such as the range of applicability of unit hydrograph method or the synthetic unit hydrograph method and so on.

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CONTENTS

1. INTRODUCTION

Since Sherman proposed the unit hydrograph method based on the three hypotheses of linearity, it has been widely used for runoff analysis in mountainous catchments because of its simplicity and applicability.

The basic principle of the unit hydrograph method is such that there exists a linear relationship between the discharge $Q(t)$ and the rainfall $r(t)$ expressed as

$$Q(t)=\int_{-\infty}^{t}X(\tau)r(t-\tau)d\tau$$

where $X(t)$ is the unit hydrograph given as a kernel of the linear integral equation. This implies that it is difficult or impossible for the classical dynamic analysis to explain the phenomena of extremely nonuniform flow field and arbitrariness such as the runoff process in mountainous catchments. Careful attentions should be paid to this approach because it implies a stochastic concept and includes an excellent idea in methodology as well as in simple applicability.

Although the rainfall movement in the mountainous catchment obviously composes a dynamic system, the exact approach can be said to be beyond the limits of classical dynamics, because the degrees of freedom are too large from the viewpoint of microscopic substance of rainfall. In fact, however, there is no need for us to take interest in completely knowing the microscopic situation of a whole system at every time, that is, describing a whole system entirely and dynamically. Thus the only goal is to gain the macroscopic knowledge of phenomena at the gauging station. It is clear that the above linear integral equation as the basis of the unit hydrograph method is derived not from the dynamic law but from the empirical fact. The advantage of the unit hydrograph method lies in the fact that the necessary information can be obtained if the theory is correct in principle without the microscopic state of a system being dynamically traced.

However, as the rainfall-runoff process is one of dynamic phenomena, it goes without saying that the significance of the unit hydrograph method will be lost unless the dynamic rationality of the law based on the above empirical facts is guaranteed. From the above arguments the main thrusts of unit hydrograph method should be discussed from the following viewpoints:

(i) The dynamic importance of unit hydrograph method should be made clear.

(ii) If the basic principle of unit hydrograph method is beyond a dynamic law, the most desirable form of the unit rainfall and unit hydrograph method have to be found depending on the problems concerned. Furthermore the errors of unit hydrograph method resulting from use of the unit rainfall and the unit hydrograph should be quantified. For this purpose, it is necessary to investigate the fundamental structures of the unit rainfall and the unit hydrograph.

The above two themes are connected directly with many problems such as the optimum elements of the unit rainfall and the unit hydrograph, the limitations for application and the synthetic unit hydrograph, which are widely discussed at present. The solutions to the questions will help overcome the difficulties involved in use of the unit hydrograph method in hydrologic practices.

The present study focuses on solving these two themes, while investigating the behavior of rainfall in mountainous catchments from a hydraulic standpoint.

2. HYDRAULIC MECHANISM OF RAINFALL-RUNOFF PROCESS

Runoff phenomena caused by rainfall in mountainous catchments is a kinematic event with the rainfall as source and the catchment as transformation. However, as the rainfall occurs randomly, its temporal and spatial distributions vary in a probabilistic way. Even though the catchment providing the field of rainfall movement is mainly divided into two elements such as the slope with spatial characteristics and the channel with linear one, these two elements are quite nonuniformly distributed. From those viewpoints, runoff phenomena can be considered as rainfall movement characterized with randomness in a nonuniform field. The extreme difficulty of theoretically seeking the universal law of rainfall-runoff process lies in the complexity of these characteristics.

In the past the rainfall-runoff mechanism in a catchment was theoretically studied by Richard[1] and Zoch[2]. Due to their relatively bold and oversimplified presupposition, they appear to have no dynamic rationale even on the qualitative account.

The present study aims to investigate the hydraulic characteristics of rainfall-runoff process to obtain a fundamental meaning of the unit hydrograph method. The rigorous theory would not be possible without the introduction of the stochastic concept along with consideration of random characteristics of runoff phenomena. The subsequent contexts of the paper are mainly focused on the surface runoff with effective rainfall, rather than on the spatial distribution and loss phenomena of rainfall.

(1) Basic equations on propagation of rainfall and discharge

When attempts are made to clarify the dynamic phenomena theoretically, the differential equations are generally applied. In a nonuniform field, the theoretical construction on the basis of microscopic equivalent concept is acceptable only in the case where the average quantity appears to represent the characteristics.

A catchment acts like the motion field in runoff phenomena and the important factors of the characteristics are the gradient to express the gravity effect and the roughness to represent the friction. Although the catchment consists of two fields of slope and channel, the difference of characteristics between the fields (e.g., roughness) is verified by the experiments of Palmer[6] and the calculated results by using a characteristic curve. Thus the slope and channel cannot be considered to be uniform as an equivalent flow field on the dynamic analysis of runoff phenomena. The division of catchment into the two fields such as the slope and channel is considered merely as extremely simplified situations. The factors of slope or channel are very nonuniform. However, the conversion of these nonuniform factors to average values is reasonably acceptable, because there is no significant difference in the runoff mechanisms and is little variance of hydrologic variables.

From the above discussions, the hydraulic mechanisms of rainfall-runoff process on both slope and channel ought to be considered. When the channel takes the lateral inflow, the equations of motion and continuity are represented on the assumption that the flow is uniform as follows:

$$A = f_n(Q) = KQ^{p} \tag{2.1}$$

$$\partial A/\partial t + \partial Q/\partial x = q(t) \tag{2.2}$$

where A is cross sectional area of flow, Q is discharge, $q(t)$ is lateral inflow per unit length on channel (a function of time), x is distance, and t is time. When the flow regime is assumed to follow Manning's resistance law, the parameters K and p are given by

$$K = \left\{n(\sin\theta)^{-1/2}K_1^{2/3}\right\}, \quad p = 3/(2z+3)$$

where n is the roughness coefficient, $\sin\theta$ is the gradient of slope, and K_1 and z are the constants when the relationship between hydraulic radius R and cross sectional area A is defined as $R = K_1 A^z$.

From eqs. (2.1) and (2.2), the characteristic equation is derived as

$$\frac{dx}{1} = \frac{dt}{dA/dQ} = \frac{dQ}{q(t)} \tag{2.3}$$

Substitution of eq. (2.1) into eq (2.4) gives

$$\frac{dx}{1} = \frac{dt}{pKQ^{p-1}} = \frac{dQ}{q(t)} \tag{2.4}$$

It follows from eqs. (2.3) and (2.4) that the equation on the characteristic curve is expressed as

$$\frac{dx}{dt} = \frac{dQ}{dA} = \frac{Q^{1-p}}{pK} \tag{2.5}$$

The integration of eq. (2.5) yields

$$Q = \left\{\int_\tau^t q(t)dt/K\right\}^{1/p} + Q(\xi,\tau)\ , \qquad Q = \int_\xi^x q(t)dx + Q(\xi,\tau) \tag{2.6}$$

where $Q(\xi, \tau)$ denotes discharge at location ξ and time τ from which the characteristic curve starts, depending on the initial and boundary conditions of discharge.

When substituting eq. (2.6) into eq. (2.5), the differential equation of characteristic curve is obtained by

$$\frac{dx}{dt} = \frac{1}{pK}\left[\left\{\int_\tau^t q(t)dt/K\right\}^{1/p} + Q(\xi,\tau)\right]^{1-p} \tag{2.7}$$

As a result of integrating eq. (2.7), the characteristic curve is derived as follows:

$$x = \frac{1}{pK}\int_\tau^t ds\left[\left\{\int_\tau^s q(z)dz/K\right\}^{1/p} + Q(\xi,\tau)\right]^{1-p} + \xi \tag{2.8}$$

The mathematical explanation on the relationship between eqs. (2.6) and (2.8) is such that, on the three dimensional space of *Q-x-t*, the characteristic equation (2.8) results from the curve included in the solution or integration surface projected to *x-t* plane and consequently the relation of eq. (2.6) is obtained on this curve. The general solution is expressed through the integration of two independent equations from eqs. (2.3) and (2.4) as the following arbitrary function:

$$Q=F\left[Q-\left\{\int q(t)dt/K\right\}^{1/p},\ x-(1/pK)\int Q^{1-p}dt\right]=0$$

Since the flow on slope is assumed to be approximated by a two-dimensional flow, the lateral inflow and the discharge on the slope can be replaced by the effective rainfall intensity $r(t)$ and q, respectively. When the initial and boundary conditions of slope flow are set as zero, the following equations are obtained in accordance with eqs. (2.1), (2.2), (2.6) and (2.8) on the channel;

$$h'=K'q^{p'} \tag{2.9}$$

$$\partial h'/\partial t+\partial q/\partial x'=r(t) \tag{2.10}$$

$$q=\left\{\int_{\tau}^{t}r(t)dt/K'\right\}^{1/p'},\quad q=\int_{\xi}^{x'}r(t)dx' \tag{2.11}$$

$$x'=\frac{1}{p'K'^{1/p'}}\int_{\tau}^{t}ds\left\{\int_{\tau}^{s}r(z)dz\right\}^{1/p'-1}+\xi' \tag{2.12}$$

where the superscript of "/ " denotes the value on the slope. If the slope flow obeys Manning's resistance law, the following relations are held:

$$K'=\left(n'/\sqrt{\sin\theta'}\right)^{1/p'},\quad p'=3/5$$

Equations (2.8) and (2.12) express propagation states of rainfall, while eqs. (2.6) and (2.11) the relations of discharge to be satisfied on the propagation equations. These equations are the fundamental ones in the proceeding analyses and herein the physical meaning of the equations is explained below.

As the discharge into the channel is assumed to vary with time but to keep steady in space, the rectangular model basin (or parallelogram) is handled for simplicity (see **Fig. 2.1**). In the case where a channel is formed amidst the catchment, $q(t)$ is considered as a combined inflow from both slopes, which is the same as the one on a single slope. The coordinate axis on the slope is defined along the average flow routing path of rainfall from the top, while that on the channel from upstream to downstream. The flow routing lengths on the slope and channel are expressed as B and L, respectively. On these coordinate planes, both ξ and ξ' in eqs. (2.8) and (2.12) become zero.

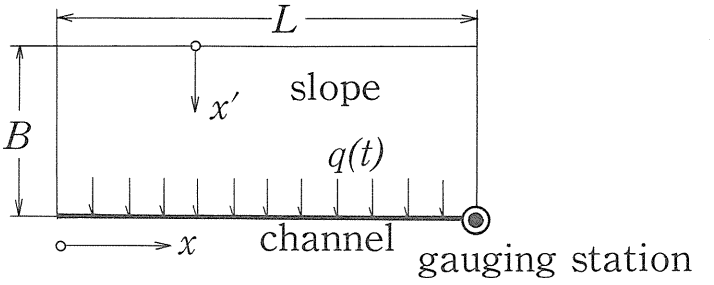

Fig. 2.1 Schematic diagram of a rectangular model basin

Figure 2.2 shows the process in which rainfall starting from the top of slope reaches the end of channel B, flows into the channel and moves down to the monitoring station at the extreme end of channel. Each characteristic curve representing rainfall propagation on the slope and channel is expressed as eqs. (2.12) and (2.8). Equations (2.11) and (2.6) demonstrate the relations of rainfall and discharge within the propagation time with inflow at the channel and the discharge at the monitoring station, respectively, as shown in the shadowed part on the figure. Four parameters of p', K', p and K for the characteristics of slope and channel represent the lag effect.

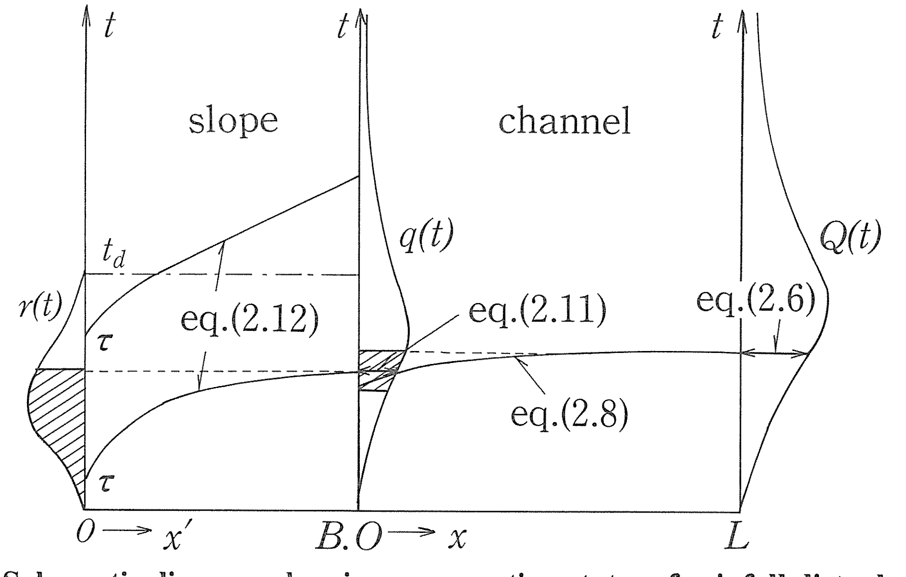

Fig. 2.2 Schematic diagram showing propagation states of rainfall disturbance and relations between equations of propagation and discharge

The reason should be explained why Manning's resistance law to the momentum equation is introduced. The flow on the slope is caused by rainfall, having the water depth of several values in the unit of "mm" on "cm" and moves down, thinly covering the soil surface . This flow is often termed thin sheet flow, shallow flow, overland flow or low flow. Reasonable among them is the low flow named by Palmer, for defining slope flow in the catchment, because the catchment slope is characterized by complex vegetation covering and its extreme nonuniformity.

The discharge flowing over the thin soil surface has been investigated by many researchers who have confirmed the flow characteristics by Reynolds number in most cases. The critical values of Reynolds number where the flow state shifts from laminar flow to turburant flow are reported as the values of 310, 300 to 330[3] or 550 to 770[4]. This value is generally taken as about 500 in the open channel flow and is rather smaller than the above values except one example. It is of interest to compare with the experimental result by Iwagaki[5], indicating that the critical Reynolds number decreases with an increase of roughness. As a result, there exist two domains of laminar and turburant flows even on the catchment slope. Because of the extreme nonuniformity on the slope, it is adequate to handle the flow as turburant one over all slopes. It is quite reasonable from the viewpoint of Palmer's conclusion[6] through large experiments in the grassland that the flow is turbulent, the discharge is proportional to the water depth with the power of 5/3 and the flow regime follows Manning's resistance law.

The friction effect is so predominant for the flow on the catchment slope that other effects are not taken into account herein (e.g., inertia term, changing rate of gradient of water surface and movement state of flow due to the inflow of rainfall and infiltration). It is allowed to adopt Manning's resistance law in the momentum equation for the channel flow in order to investigate the behavior in the vicinity of flood peaks. From an engineering point of view, the prediction of peak discharge is the most important issue for runoff phenomena. It is theoretically pointed out in the subsequent section that the slope is the most predominant catchment characteristics to largely affect the runoff hydrograph. Therefore, the error resulting from use of Manning's law appears to be negligibly small in runoff problems.

The validity of the above assumptions and approximation is conformed to the study by Sueishi[7] who applied the equations of the characteristic curve to the historical floods in the Odo River and the Yura River with consequence of good results.

(2) Relationship between the characteristics of runoff, rainfall and catchment

The basic equations developed in the preceding section is used to clarify the relationships between some significant characteristics of runoff, rainfall and catchment herein. Those results will become the basis of theoretical analyses for the unit hydrograph method described later.

(a) On propagation time of rainfall disturbance Propagation time discussed here is defined as the propagation of disturbance generated with input of rainfall, which does not mean the propagation of water particle. Physically, it might be understood as advection of wave motions. Under the assumptions that the two-dimensional flow follows Manning's resistance law and the influence of flow movement due to the lateral inflow is negligible, the relationship between propagation velocity dx/dt of rainfall disturbance and average velocity u is represented as

$$dx/dt = 5u/3$$

when assuming Chezy's resistance law, the right-hand term of the above equation is expressed as $3u/2$. On the other hand, the following equation is derived from eq. (2.5) as

$$dx/dt = dQ/dA$$

This equation is consistent with Kleitz-Seddon's law describing the propagation velocity of flood peak. Although the disturbance defined herein is slightly different from the propagation of flood flow surrounded with the embankments without lateral inflow. However, its mathematical meaning is very clear: dx/dt is a tangent to the characteristic curve which is obtained by the projection of the curve included in the integrated surface Q(or q) to the x-t plane, which is derived from eqs. (2.1) and (2.2) (or (2.9) and (2.19)). As the relation of eq. (2.6) (or (2.11)) can hold on eq. (2.8) (or (2.12)) of the characteristic curve, the concept of concentration time of rainfall must be considered as the propagation time of disturbance. Therefore, the definition of concentration time with use of velocity by Richard or Zoch is not reasonable. From a viewpoint of hydraulic significance, the propagation time as described below is represented as the concentration time of rainfall.

Now let us consider the propagation time t_B of flow on the catchment slope, because rainfall in the catchment flows into channel after moving down on the slope and reaches the gauging station. The relationship between the propagation time $(t-\tau)$ and hydrologic quantity is represented from eq. (2.12) as

$$p'BK^{1/p'} = \int_\tau^t ds \left\{ \int_\tau^s r(z)dz \right\}^{1/p'-1} \tag{2.13}$$

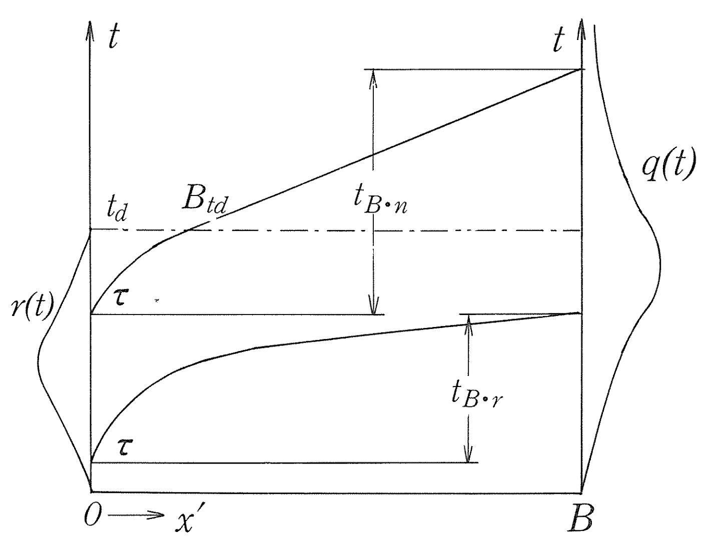

Fig. 2.3 Characteristic curves showing propagaiton states of rainfall disturbance on side slope suface in the case of $t_d > t_{B \cdot r}$

The flow mechanism on the slope with rainfall differs significantly from that without rainfall. Thus, let us first consider the particular case where the propagation of rainfall disturbance covers all the slope within a rainfall duration. For this case, the propagation time is denoted by $t_{B\cdot r}$. The right-hand side of eq. (2.13) is integrated by using average rainfall intensity r_m within a propagation time as follows:

$$\int_{\tau}^{t} ds\left\{\int_{\tau}^{s} r(z)dz\right\}^{1/p'-1} = \int_{\tau}^{t}\left[r_m(t-\tau)+\left\{R(t)-R(\tau)-r_m(t-\tau)\right\}\right]^{1/p'-1}dt \tag{2.14}$$

where

$$R(t)=\int_0^t r(t)dt, \qquad r_m=\int_{t_{B\cdot r}} r(t)dt \Big/ \int_{t_{B\cdot r}} dt$$

The second term of the right-hand side in the above equation is replaced by

$$\left\{R(t)-R(\tau)\right\}-r_m(t-\tau)=N(t,\tau) \tag{2.15}$$

$N(t,\tau)$ denotes the shape effect of hyetograph within the propagation time and the integrated result is shown in the shadowed portion of Fig. 2.4. The sign of $N(t,\tau)$ changes with the sign of the first derivative of rainfall $r(t)$ within the propagation time as follows:

$$N(t,\tau)\le 0 \quad for \quad dr/dt\ge 0 \quad and \quad N(t,\tau)>0 \quad for \quad dr/dt<0 \tag{2.16}$$

Since $N(t,\tau)$ is much smaller than the rainfall intensity $r_m(t-\tau)$ involved in the first term of the integrated part, eq. (2.14) is expressed with high accuracy as

$$p'BK'^{1/p'}=\int_0^{t-\tau}(r_m t)^{1/p'-1}dt \tag{2.17}$$

This equation can easily be integrated, giving rise to the relationship between propagation time, average rainfall intensity and catchment characteristics as follows:

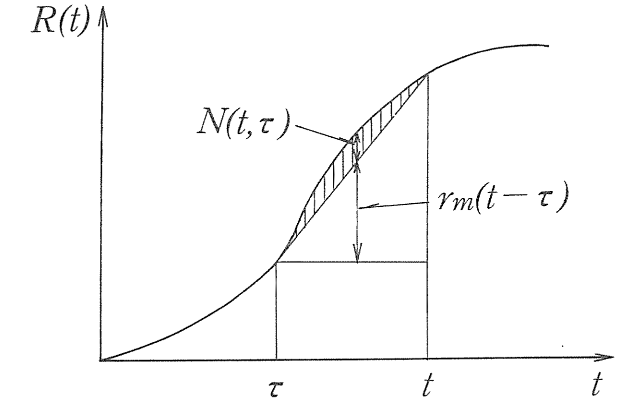

Fig. 2.4 Relation between rainfall hourly distribution and $N(t,\tau)$

$$t_{B\cdot r}=t-\tau=K'B^{p'}\Big/r_m^{1-p'} \tag{2.18}$$

Equation (2.18) is derived from the assumption of $N(t, \tau)=0$ and hence there is a difference between the actual propagation time $t_{B \cdot s}$ and approximate one $t_{B \cdot r}$. This difference is expressed by

$$t_{B \circ r} \leq t_{B \circ s} \quad for \quad dr/dt \geq 0 \quad and \quad t_{B \circ r} > t_{B \circ s} \quad for \quad dr/dt < 0 \tag{2.19}$$

Therefore, the propagation time is affected by a changing rate dr/dt of hyetograph even if rainfall intensities are identical within the propagation time. Generally, the propagation time for an increase of rainfall intensity is larger than that for a decrease. However, as the term $N(t, \tau)$ is extremely smaller than the term $r_m(t-\tau)$, eq. (2.18) has a high accuracy in which the effect of temporal distribution of rainfall is neglected. Moreover, in the vicinity of the time to peak rainfall, the relation of $r(\tau_p)=r(t_p)$ can hold between starting time τ_p and arriving time t_p for the propagation to generate the maximum discharge and consequently the integral value of $N(t, \tau)$ becomes nearly zero, resulting from counterbalance of the effects of rainfall distribution between τ_p and t_p.

Let us next consider propagation time $t_{B \cdot n}$ for the case where the rain stops before the rainfall disturbance starting from the top of slope does not reach the channel. When B_{td} denotes the traveling distance of rainfall from starting time τ to ceasing time t_d of rainfall as shown in **Fig. 2.3**, the following equation is obtained through eq. (2.17), neglecting the rainfall distribution as:

$$p' B_{td} K'^{1/p'} = \int_0^{t_d-\tau} (r_m t)^{1/p'-1} dt = p r_m^{1/p'-1} (t_d-\tau)^{1/p'} \tag{2.20}$$

where

$$r_m = \int_\tau^{t_d} r(t)dt \Big/ \int_\tau^{t_d} dt$$

The propagation velocity for B_{td} is obtained through eq.(2.12) as follows:

$$\left(\frac{dx'}{dt}\right)_{t=td} = \frac{1}{p'K'^{1/p'}} \left\{\int_\tau^{t_d} r(t)dt\right\}^{1/p'-1} = \frac{\{r_m(t_d-\tau)\}^{1/p'-1}}{p'K'^{1/p'}} \tag{2.21}$$

The propagation time $(t-t_d)$ taken from B_{td} to the end of channel B is calculated by

$$t-t_d = \frac{B-B_{td}}{(dx'/d)_{t=td}} = (B-B_{td}) \frac{p'K'^{1/p'}}{\{r_m(t_d-\tau)\}^{1/p'-1}} \tag{2.22}$$

When cancelling out B_{td} from eqs. (2.21) and (2.22), the following relationship between characteristics of rainfall τ and t and catchment is obtained:

$$(t_d-\tau)^{1/p'-1}\{(t-t_d)+p'(t_d-\tau)\} = p'BK'^{1/p'} / r_m^{1/p'-1} \tag{2.23}$$

Although it is difficult to explicitly derive the propagation time $t_{B \cdot n}=t-\tau$ from the above equation, the time to peak inflow $q(t)$ occurs between τ which is the arriving time of disturbance at $\tau=0$ and t_d (see

Fig. 2.5). For practical significance, it is important to investigate the case of $\tau=0$. If the propagation time for this case is expressed as $t_{B\cdot n\,\tau=0}$, the following relation is directly derived from eq. (2.23):

$$t_{B\cdot n\tau=0}=\frac{p'B(K')^{1/p'}}{R_{td}^{1/p'-1}}+(1-p')t_d \tag{2.24}$$

where

$$R_{td}=\int_0^{t_d} r(t)dt$$

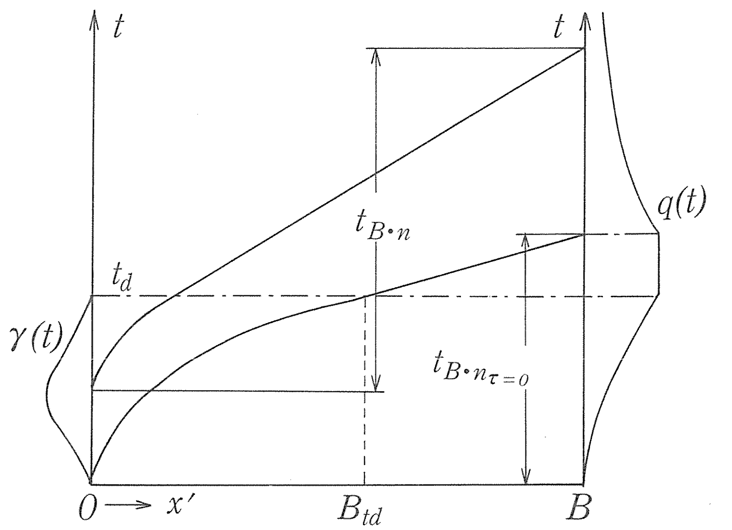

Fig. 2.5 Characteristic curves showing propagation states of rainfall disturbance on side slope surface in the case of $t_d<t_{B\cdot r}$

The following relationship on the channel is found between propagation time $t_L=t-\tau$ from the upstream to the downstream, the lateral inflow $q(t)$, inflow $Q(0,\tau)$ from the upstream end of channel and catchment characteristics as

$$pLK=\int_\tau^t ds\left[\left\{\int_\tau^s q(z)dz/K\right\}^{1/p}+Q(0,\tau)\right]^{1-p} \tag{2.25}$$

This equation shows that propagation state of rainfall in the channel is controlled by two factors such as lateral inflow $q(t)$ and inflow $Q(0,\tau)$ from the upstream end of channel. $Q(0,\tau)$ can generally be set as zero except for the special case. $q(t)$ always exists in the flood runoff caused by heavy rainfall in a short term. The shape effect of time distribution $q(t)$ can be negligible, because $q(t)$ becomes quite flatter than $r(t)$ mainly due to the lag effect on the slope. As a result, the following relation is derived through the same procedure as in eq. (2.18):

$$t_L = KL^p / q_m^{1-p} \tag{2.26}$$

where

$$q_m = \int_{t_L} q(t)dt \Big/ \int_{t_L} dt$$

When the rainfall-runoff process is numerically traced in a real river catchment by using the proposed theory, it is necessary to divide the whole catchment into several subcatchments[7]. In such situations, the inflow $Q(0, \tau)$ from the upstream end of each subcatchment becomes extremely large and the lateral inflow can be negligible. The propagation time t_L in the channel of divided subcatchments is represented, when ignoring $q(t)$ in eq. (2.25), as follows:

$$t_L = pLKQ(0,\tau)^{p-1} \tag{2.27}$$

The calculation methodology is omitted herein, because it is beyond the scope of this paper.

(b) Comparison for the effects of slope and channel on runoff The effect of catchment characteristics on runoff has been already discussed in detail with methodology of characteristic curves and it has been verified that the slope gives much larger effect than the channel in the mountainous catchment[8]. The above statement can be approached analytically in the present study.

The runoff characteristics is mainly controlled by the propagation time, which not only represents the phase difference of time lag but also specifies the discharge volume. The discharge Q at a gauging station is expressed as

$$Q = \int_{t_L} \frac{\int_{t_B} r(t)dt}{t_B} \cdot dt \cdot \frac{BL}{t_L}$$

B, L and $r(t)$ dominate the propagation time t_B and t_L and consequently Q is controlled by t_B and t_L.

The hydrograph is regarded as a kind of images of hyetograph and the relation of this image is controlled by the propagation time. From this argument, the effects of catchment characteristics on runoff phenomena are comprehensively represented by the propagation time. In both cases where t_B and t_L are zero, the catchment characteristics has no effect because hyetograph is converted into the hydrograph , while keeping the same shape. An increase of propagation time makes larger the distortion effect on the image to discharge, yielding an increase of the effect of catchment characteristics. Therefore, it suffices to analyze each propagation time by comparing the effects of slope and channel on runoff.

This paper deals with the special case where the propagation of rainfall disturbance covers all slopes during rainfall and the assumption appears to be valid for flood runoff. From eqs. (2.18) and (2.26), the propagation times t_B and t_L are given by

$$t_B=K'B^{p'}/r_m^{1-p'} \quad (2.28)$$

where

$$r_m=\int_{t_B} r(t)dt \Big/ \int_{t_B} dt$$

$$t_L=KL^{p}/q_m^{1-p} \quad (2.29)$$

where

$$q_m=\int_{t_L} q(t)dt \Big/ \int_{t_L} dt$$

However, as the rainfall flows into the channel after first moving down on the slope and reaches the gauging station along the channel, the comparison of t_B and t_L ought to be made according to the same time periods of rainfall as shown in **Fig. 2.2**. In this case, eq. (2.11) leads to the following relation:

$$q_m=r_m B \quad (2.30)$$

The following equation is obtained via use of three equations (2.28) to (2.30):

$$S_t\equiv\frac{t_L}{t_B}=\frac{KL^{p}}{K'B^{p'-(p-p')}}r_m^{p-p'} \quad (2.31)$$

where p' and p are the values for the relationship between the cross sectional area and discharge on the slope and the channel, respectively. Under the assumption that both flows conform to Manning's resistance law, p' is almost equal to p. The observed records[7], [9] show that $p\geq p'$ in most cases and the difference $p-p'$ is close to 0.1. When the slope flow is two-dimensional and the channel shape is assumed to be rectangular, the values of p' and p are equal to 0.6. Hence, eq. (2.31) is approximated by

$$S_t=KL^{p}/(K'B) \quad (2.32)$$

where $p'=p$ is assumed.

Note from eq. (2.32) that S_t does not depend on rainfall characteristics, but only on catchment characteristics. Moreover, it is very clear that S_t has much more hydraulic significance than Horton's distribution coefficient $F=B/L$, because it is a nondimensional number which expresses the quantitative relationship between catchment characteristics and runoff process.

Generally, the value of K' in a natural catchment is several times as large as that of K[7], [9]. If $K'=5K$, B=1,000m, L=20,000m and p=0.6, S_t is nearly equal to 1/10 from eq. (2.32). It can be understood that the value of S_t in a natural catchment ranges in a few decimal fraction. For the particular case of S_t=1, the effect of slope on the lag time transformed from rainfall to discharge is equal to that of channel. The smaller S_t, the larger is the effect of slope than that of channel. This result clearly shows that the effect of slope is quite larger than that of channel.

When comparing the flood runoffs with identical catchment areas, it can be proven that the arriving time from the top of slope to the downstream decreases with an increase of *L/B*. However, it is too hasty to judge flood propagation only by the value of *L/B*, because the average rainfall intensity also decreases at the same time.

<u>(c) Occurrence condition of peak discharge</u> Herein an interest is centered on what portion of hyetograph contributes to the generation of peak discharge. For this objective, the relationship will be found between the starting and arriving times of propagation. It is found from the previous discussion that the effect of propagation in the channel on runoff is negligibly small. Thus, the runoff mechanism at the gauging station can almost be determined, given the approximate condition on inflow hydrograph from slope to channel.

The inflow $q(t)$ from slope to channel is expressed via eq. (2.11) as

$$q(t)=\left\{\int_{\tau}^{t} r(t)dt/K'\right\}^{1/p'} \tag{2.33}$$

$$q(t)=r_m B \tag{2.34}$$

Substituting eq. (2.18) into eq. (2.34) gives

$$q(t)=\{BK'/(t-\tau)\}^{1/(1-p')} \tag{2.35}$$

Then differentiating eqs. (2.33) and (2.35) by t and setting them to zero yield the occurrence condition of maximum discharge as follows:

$$r(t)-\frac{d}{dt}\int_{0}^{\tau} r(t)dt=0 \tag{2.36}$$

$$\frac{d}{dt}(t-\tau)=0 \tag{2.37}$$

Equation (2.37) demonstrates that the propagation time of rainfall generating maximum discharge takes the minimum and differentiations in terms of t and τ are identical. It follows from eq. (2.36) that the rainfall intensity at starting time τ_p generating peak discharge is equal to that at arriving time t_p as

$$r(\tau_p)=r(t_p) \tag{2.38}$$

Equation (2.38) indicates that the rainfall within a period having an identical intensity before and after the peak rainfall has an influence on the maximum discharge. If this relation can hold at a gauging station with no loss event considered, the propagation time $(t_p-\tau_p)$ of peak discharge is easily calculated from the observed data of rainfall and discharge as shown in **Fig. 2.6**.

As eq. (2.38) is realized on the inflow into channel in a strict sense, it is required to know how large is the propagation error of peak discharge gained through the procedure of **Fig. 2.6**. It is apparent that arriving time t_p of propagation occurs at the time of maximum discharge and the error results from starting time τ_p. Rainfall intensity $r(\tau_{ps})$ at starting time τ_{ps} of propagation of peak discharge is generally larger than $r(\tau_p)$. Thus, $(t_p - \tau_p)$ in **Fig. 2.6** is a little bigger than the actual propagation time

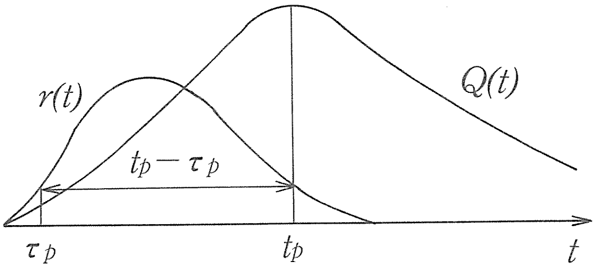

Fig. 2.6 Relation between $(t_p - \tau_p)$ and curves of rainfall and discharge

$(t_p - \tau_{ps})$, because propagation time t_L in the channel must be taken into consideration. When its relation is set as

$$(t_p - \tau_p) = (t_p - \tau_{ps}) + lt_L \tag{2.39}$$

l, which changes depending on the rainfall distribution between t_p and τ_p, is normally larger than 0 and is nearly equal to 1/2 in the case where temporal rainfall distributions for period t_L just after τ_p and period t_L just before t_p are identical*. In addition, the following relation is assumed:

$$t_p - \tau_{ps} = t_B + mt_L \tag{2.40}$$

As m varies depending on the distribution of $q(t)$ within period t_L and $q(t)$ becomes flatter than $r(t)$, m is considered to be 1/2**.

From the above two equations, the rate of lt_L to $(t_p - \tau_p)$ is obtained by

$$\sigma \equiv lt_L/(t_p - \tau_p) = lS_t/\{1 + (l+m)S_t\} \tag{2.41}$$

and the actual propagation time $(t_p - \tau_{ps})$ is formulated as follows:

$$t_p - \tau_{ps} = (t_p - \tau_p)(1 - \sigma) = (t_p - \tau_p)[1 - lS_t/\{1 + (l+m)S_t\}] \tag{2.42}$$

Therefore, when the effect of propagation time t_L in the channel is predominant, the modification of eq. (2.42) can be made by setting l and m as 1/2. For a small catchment like that in Japan, σ is very

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* In this case, $l = (1/2)(r_{mtL}/r_{mtL'}) \gtrsim 1/2$ is proved by the occurrence conditions of peak inflow on the slope and the maximum discharge on the channel, and eq. (2.11) on the inflow and eq. (2.18) on the propagation. Here, r_{mtL} is an average rainfall intensity for period t_L and $r_{mtL'}$ is an average rainfall intensity for t_L' from inflow peak to t_p. Both values are approximately identical, even though $r_{mtL} \gtrsim r_{mtL'}$.

** Since $m = t_L'/t_L$, $m = l$ can be verified under the assumption that the rainfall distribution for period t_L just after τ_p is the same as that for period t_L just before t_p as described above. However, this assumption in terms of l is strongly constrained while m is not. Even though such an assumption is not satisfactory in most cases, the value of m does not largely deviate from 1/2.

small because the value of S_t in a natural channel has a small decimal fraction and consequently eq. (2.38) is regarded as the occurrence condition of maximum discharge at the gauging station.

(d) Synthesis of elements on slope and channel Because rainfall movement on the slope and channel is quite different from each other, it is necessary to handle the elements separately. However, the elements in each process have completely the same physical meaning. For instance, the surface flow consists of the elements such as flowing distance, gradient and roughness. For the objective of this paper, individual elements on the slope and channel are lumped to synthetic variables in a single catchment, which give the same hydrologic characteristics. The following notations are used to represent the synthetic variables with subscript of "0".

L_0 : synthetic factor of B and L for flowing distances

$sin\theta_0$: synthetic factor of $sin\theta'$ and $sin\theta$ for gradients

n_0 : synthetic factor of n' and n for coefficients of roughness

K_0 : synthetic factor of K' and K for parameters of catchment

Although catchment characteristics is represented by those synthetic factors as discussed below, their relationships and characteristics on the slope and channel are too complicated to be added or averaged in a simple way. Conventional studies by Richard and others have equivalently handled all slopes and channels without distinction and the rainfall is assumed to directly flow down from the landing point to the gauging station. Such approaches appear to make it easy to determine synthetic factors. Actually, these assumptions are not reasonable. This is a main reason why the concept of synthetic factors is applied in this study and its significance must be approached from a viewpoint of relations with runoff mechanism. The empirical formula of Snyder[10] and Nakayasu[11] on the lag time of maximum discharge is noteworthy, because the effects of flowing distance are introduced as geometric average process between the distance from the gauging station to the farthest point on the channel and the distance to the gravity center of catchment, or geometric average between the former and the distance of the point where a catchment width becomes maximum.

A study on the relationship between synthetic factors and elements of slope and catchment is an important subject in the future. A suggestion to this issue is given by the ratio S_t with respect to the propagation time of channel to that of slope as mentioned before. From the analyzed results, synthetic factors can mainly be dominated by the elements on the slope.

3. DYNAMIC SIGNIFICANCE OF THE UNIT HYDROGRAPH METHOD AND LINEARITY OF RUNOFF PHENOMENA

(1) Dynamic significance of the unit hydrograph method

This chapter discusses dynamic conditions on which the unit hydrograph theory bases . In the following context, the catchment parameters are represented by a set of synthetic variables as described in previous chapter, and the exponents of p and p' involved in the momentum equations are assumed to take the same value.

The synthetic element is mainly governed by elements of the slope as mentioned above. However, the extreme inhomogeneity of slope geomorphology makes it hard to represent the paths of rain water on the slope in a strict sense. From a viewpoint of qualitative comparison between the unit hydrograph method derived from the dynamic law and the actual runoff phenomena determined by a different law from an idealized event, the path of rain water in both cases could be considered the same. Let x_0 be the synthetic flow distance along the flow routing path. This coordinate system does not mean a straight line on which rain water flows down to the gauging station from an arbitrary point in the catchment. It indicates a continuous curve which is orthogonal to the contour line representing various flow routing paths. Such a concept does not necessarily distinguish between the flow routing path on the slope and the channel. It is found from this assumption that the same result of runoff phenomena at the gauging station can be expected by using suitable synthetic elements as if the routing paths on the slope and channel are treated distinctly. Under this coordinate system, the rainfall can be regarded as the inflow to a whole catchment.

Therefore, the characteristic curve in the synthetic runoff system is expressed as

$$\frac{dx_0}{dt} = \frac{1}{pk_0^{1/p}}\left\{\int_r^t r(t)dt\right\}^{1/p-1} \tag{3.1}$$

or

$$x_0 = \frac{1}{pK_0^{1/p}}\int_r^t ds\left[\int_r^s r(z)dz\right]^{1/p-1} \tag{3.2}$$

Then, the discharge along the characteristic curve is given by

$$Q = \left[\int_\tau^t r(t)dt/K_0\right]^{1/p} \tag{3.3}$$

The discharge Q is given along the flow distance x_0. The actual discharge at the gauging station can be obtained by integrating the total discharge along all flow directions as mentioned above.

When the flow is assumed to be two dimensional in the synthetic runoff system, the equation of motion is expressed as

$$h = K_0 Q^p \tag{3.4}$$

where p is 0.6 in Manning's resistance law and the selection of 0.6 is supported by Palmer's experimental study[6].

The unit hydrograph method is recognized as a synthetic expression of discharge hydrograph representing the runoff response of lag characteristics to rainfall in a catchment and is based on the following three hypotheses:

i) The rainfall excess is assumed to be uniform over the catchment. A pattern of runoff hydrograph is identical, when produced by a rainfall event with equal intensity and duration.

ii) The temporal distribution of runoff does not change for the runoff response to rainfall events with varying intensities and equal duration and changes the patterns in proportion to rainfall intensities.

iii) The above two hypotheses can always be valid for a single-period storm as well as milti-period storms.

These three hypotheses might be examined from a dynamic viewpoint. The most important property of the first hypothesis is that the pattern of runoff hydrograph is determined only by the rainfall condition. It is apparent that the patterns of runoff even for the same rainfall conditions may be different from each other in a catchment, because the conditions of vegetation and moisture content on the slope are not the same in a year. For such a case, it is preferable to treat these catchment conditions from a hydrologic viewpoint. Then the first hypothesis appears to be acceptable.

The second and the third hypotheses are entirely equivalent to each other and are based on the linearity of phenomenon. In other words, the second hypothesis that discharge is proportional to rainfall intensity is based on the principle of superposition, and corresponds to the condition of p=1 in eq. (3.3). The third hypothesis that the propagation characteristics of rain disturbance is not affected by the rainfall condition is equivalent to p=1 in eq. (3.1) or (3.2). In such a case, the propagation characteristics is obviously given only by the catchment characteristics K_o, and the propagation velocity is expressed as follows:

$$\frac{dx}{dt} = 1/K_0 = \sqrt{\sin\theta_0}/n_0$$

which is the same as Manning's formula without considering the effect of water depth.

It is obvious from the above discussion that the assumption of unit hydrograph method is realized in a case where the momentum equation is linear, that is, p=1. Therefore the second and third hypotheses denote the assumption of linearity. Tategami[12] has discussed the necessary conditions of the unit hydrograph method, using Horton's storage equation for the recession characteristics of runoff hydrograph. Since the assumption of unit hydrograph method is based on the linearity of momentum equation, the present study can also include the method proposed by Tategami.

In general, the value of p is not unity in actual runoff phenomenon. Therefore the assumption of unit hydrograph method is not satisfied in a strict sense. However, the unit hydrograph method is widely used as one of the most effective methods for runoff analysis. For example, Zoch's theory[2], which is famous for theoretical treatment of runoff problems, bases on the assumption of "Discharge is proportional to the catchment storage at any time" or "Discharge at the end of a catchment is proportional to the flow water depth". This assumption allows one to use the Hortonian storage equation, the linearity of momentum equation and the unit hydrograph method itself. Thus, Zoch's theory does not deem to have higher-order dynamic meanings than the unit hydrograph method. Furthermore the unit hydrograph method has such an excellent concept that the runoff response characteristics even in an extremely heterogeneous catchment can be expressed by use of a unit hydrograph. The unit hydrograph method is much superior to Zoch's theory in terms of simplicity and basic understanding of actual runoff phenomena, even though the dynamic significance is not yet clarified .

(2) Linearity of runoff phenomena

As already stated above, it is needless to say that applicability of unit hydrograph method depends on the linearity of runoff phenomena, because it is based on the assumption of linearity. In the past there were some doubts about the applicability of unit hydrograph method to runoff analysis in Japanese river catchments with high nonlinearity. In order to discuss the practical engineering problems such as the accuracy and the limitation of applicability of unit hydrograph method, the linearity of runoff phenomena is examined in the following contexts.

Since rainfall and catchment characteristics are two essential factors in runoff phenomena, it is needed to investigate the relationship between these two factors and lineality of phenomena. When the dynamic behavior is rapid and the differential coefficients relating to the physical quantity in time and space are large, then the degree of nonlinearity increases and it is anticipated that the accuracy of unit hydrograph method based on the linearity of phenomena might decrease.

The behavior of water movement at any point and the linearity of momentum equation are controlled by two factors; the first is rainfall characteristic such as average intensity and temporal distribution of rainfall, and the second is catchment characteristics such as gradient and friction roughness. On the other hand, the rainfall duration, catchment area and its shape are not related with the linearity of momentum equation. In other words, the rainfall duration represents the time field of phenomena, while the catchment area and its shape the space field. These quantities have no relevance to the linearity of runoff phenomena. It is well known from the experimental studies that the runoff phenomena in mountainous catchments affected by the temporal and spatial fields are highly correlated with rainfall conditions and catchment elements which directly control the linearity of phenomena.

When an attempt is made to understand an overall runoff phenomenon, the extent of sizes of these fields exerts a significant influence on the accuracy of unit hydrograph method based on the principle of superposition. These problems are also related with the degree of approximations in terms of superpositions in connection with rainfall and catchment characteristics. The accuracy of unit hydrograph method might increase or decrease, depending on the factors which control the linearity and nonlinearity of phenomena. The effect of the two fields on the accuracy of unit hydrograph cannot simply be discussed.

4. RELATIONSHIP BETWEEN ACCURACY OF THE UNIT HYDROGRAPH METHOD AND CHARACTERISTICS OF RAINFALL AND CATCHMENT

It is obvious from the above explanation that the average intensity and time variation of rainfall, and gradient and roughness of catchment slope affect directly the linearity of runoff phenomena as well as influence the accuracy of unit hydrograph method. When the runoff behavior changes abruptly, the average intensity and time variation of rainfall are large, roughness is small and the gradient of catchment slope is steep, the degree of nonlinearity increases. As a result, the accuracy of unit hydrograph method appears to decrease. However, it is not easy to discuss the accuracy of unit hydrograph method only from the relationship between the size and shape of catchment and rainfall duration. The accuracy of unit hydrograph method is examined by considering the interaction between the size and shape of the runoff field and rainfall and catchment characteristics which govern the linearity of basic equations.

(1) Expression of propagation velocity and discharge in the unit hydrograph method

According to the assumptions of unit hydrograph method, the propagation velocity of rainfall water, ω_u is held constant regardless of the rainfall condition in a catchment. When using eq. (3.1), ω_u can be expressed as

$$\omega_u = \left(1/pK_0^{1/p}\right)\alpha_t \quad (4.1)$$

where α_t given by

$$\alpha_t = \left\{\int_\tau^t r(t)dt\right\}^{1/p-1} \quad (4.2)$$

The unit hydrograph method assumes that the discharge Q_u is proportional to the rainfall intensity, which is actually treated as a rainfall amount in a unit time increment. When using eq. (3.3), Q_u is given by

$$Q_u = \alpha_q / K_0^{1/p} \int_\tau^t r(t)dt \quad (4.3)$$

where α_q is rewritten as

$$\alpha_q = \left\{\int_\tau^t r(t)dt\right\}^{1/p-1} \quad (4.4)$$

Hence, the following relationship is obtained between the propagation velocity in the unit hydrograph method and the two coefficients of α_t and α_q:

$$\alpha_t = \alpha_q \quad (4.5)$$

From eqs. (4.1) to (4.5), and the assumption of unit hydrograph method, the coefficients α_t or α_q must be constant in a catchment without regard to rainfall variation. However, eq. (4.2) or (4.4) varies with temporal distribution of rainfall, and also includes the propagation time $(t-\tau)$, depending upon both characteristics of rainfall and catchment. It is, therefore, obvious that catchment characteristics affects indirectly the magnitude of variation in terms of α_t or α_q, resulting in governing the precision of unit hydrograph method. Since there is the relationship of eq. (4.5) between α_t expressing the variation effect of propagation and α_q representing the variation effect of discharge, the assumption of unit hydrograph method with respect to the propagation is related with the assumption of runoff with no influence by rainfall or catchment characteristics. This fact might give an important basis on the synthetic unit hydrograph method, because the assumption of unit hydrograph method which computes the ordinate of the unit hydrograph in proportion to rainfall intensity, reveals that α_t and α_q do not change even for varying rainfall. If either of α_t and α_q is constant, the other should also be constant from eq. (4.5). These conditions make the two assumptions on the propagation and discharge of unit hydrograph method satisfied at the same time.

The assumption is not virtually realized , when α_t and α_q vary with rainfall conditions and the extent of the variation is controlled by the catchment conditions. The qualitative relationship is discussed between the extent of variations of α_t and α_q and conditions of rainfall and catchment. It is sufficient to treat α_t alone here because α_t is equal to α_q.

(2) Variation of α_t in the plane of $x_0 \sim t$

In general, the variation of α_t on the plane of $x_0 \sim t$ is a function of τ, t and x_0 as expressed in eq. (4.2). The variation of α_t for fixed τ on the characteristic curve is given by

$$\frac{d\alpha_t}{dt}=\left(\frac{1}{p}-1\right)\left\{\int_\tau^t r(t)dt\right\}^{1/p-2} r(t)\geq 0$$

As shown in **Fig. 4.1**, $d\alpha_t/dt$ takes a positive value for $r(t)>0$, and it becomes zero after the rain stops and α_t is constant. In the case where τ is not fixed, the variation of α_t is affected by τ. Then, the variation of α_t with respect to time under the condition of fixed x_0 is expected as

$$\frac{\partial\alpha_t}{\partial t}=\left(\frac{1}{p}-1\right)\left\{\int_\tau^t r(t)dt\right\}^{1/p-2}\left\{r(t)-\frac{\partial}{\partial t}\int_0^\tau r(t)dt\right\}$$

The time variation of α_t, therefore, increases or decreases depending on positive or negative values in the third parenthesis of the above equation.

Runoff mechanisms are divided into the following two cases of (a) and (b) according to catchment scale L_0 and rainfall duration t_d.

(a) In the case where the standard characteristic curve (standard characteristic curve is denoted by "s.c." hereafter) starting at $\tau=0$ and $x_0=0$ reaches the end of stream while it is raining (i.e., small L_0 or large t_d), the maximum discharge occurs during the rainfall duration.

(b) In the case where "s.c." reaches the end of stream after the rain stops (i.e., large L_0 or small t_d),

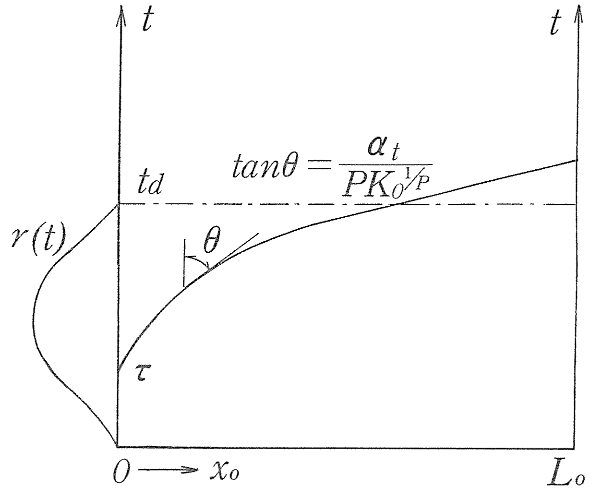

Fig. 4.1 Variation of α_t on a characteristic curve

the maximum discharge occurs after the rain stops.

For the first case of (a), when the characteristic curve starts at $\tau=0$, $\partial\alpha_t/\partial t>0$ because $\partial/\partial t \int_0^\tau r(t)dt=0$, and then α_t increases monotonously until the standard characteristic curve. $\partial\alpha_t/\partial t$ takes positive or negative values according to the sign of term $\{r(t)-\partial/\partial t \int_0^\tau r(t)dt\}$ between "s.c." and t_d, and then the variation of α_t is controlled by the temporal variation of rainfall. After t_d, $\partial\alpha_t/\partial t$ becomes negative because of $r(t)=0$ and $\partial/\partial t \int_0^\tau r(t)dt>0$, and α_t decreases monotonously. **Fig. 4.2** shows a schematic diagram as described above.

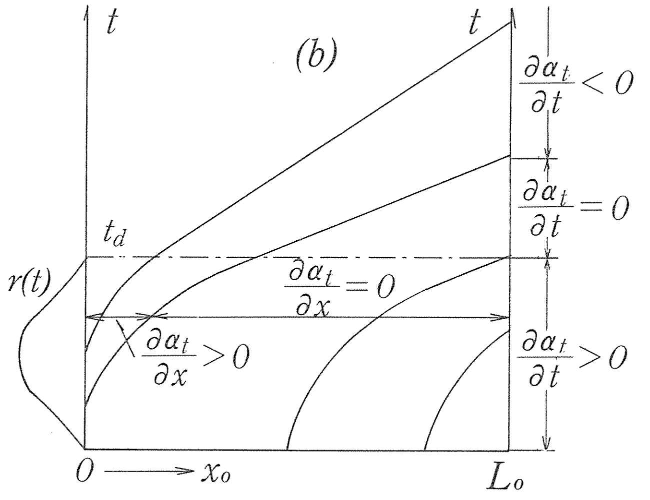

Fig. 4.2 Variation of α_t on x_0~t plane in the case of (a)

For the second case of (b), the variation of α_t is examined as follows; $\partial\alpha_t/\partial t>0$ because $r(t)>0$ and $\partial/\partial t \int_0^\tau r(t)dt=0$ and α_t increases monotonously until t_d. $r(t)=0$ and $\partial/\partial t \int_0^\tau r(t)dt=0$ between t_d and s.c., and hence $\partial\alpha_t/\partial t=0$ and α_t becomes constant. After "s.c.", $r(t)=0$, $\partial/\partial t \int_0^\tau r(t)dt>0$ and then $\partial\alpha_t/\partial t<0$ and α_t decreases monotonously. **Fig. 4.3** also schematically shows the relationship as explained above.

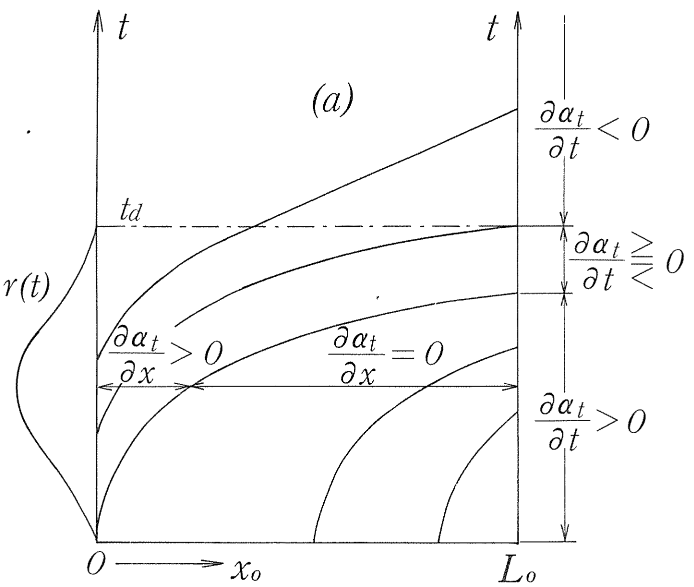

Fig. 4.3 Variation of α_t on x_0~t in the case of (b)

Furthermore, the variation of α_t with respect to x_0 is given by

$$\frac{\partial\alpha_t}{\partial x_0}=\frac{\partial\alpha_t}{\partial t}\frac{\partial t}{\partial x_0}+\frac{\partial\tau}{\partial\alpha_t}\frac{\partial\tau}{\partial x_0}=\left(\frac{1}{p}-1\right)\left\{\int_\tau^t r(t)dt\right\}^{1/p-2}\bullet\frac{\partial}{\partial\tau}\int_\tau^t r(t)dt\bullet\frac{\partial\tau}{\partial x_0}$$

Now it is necessary to examine the sign of terms $\partial/\partial\tau\int_0^\tau r(t)dt$ and $\partial\tau/\partial x_0$. As for the variation of α_t with respect to time t, variations of α_t in terms of x_0 are divided into the two cases, depending on L_0 and t_d. $\partial/\partial\tau\int_0^\tau r(t)dt<0$ and $\partial\tau/\partial x_0<0$ can be held between $x_0=0$ and "s.c.". Therefore, $\partial\alpha_t/\partial x_0>0$ and α_t increases monotonously in the x_0 direction. In addition, $\partial/\partial\tau\int_\tau^t r(t)dt=0$ and $\partial\tau/\partial x_0=0$ between "s.c." and L_0, and hence $\partial\alpha_t/\partial x_0=0$ and α_t becomes constant in the x_0 direction.

(3) Relationship between variation α_t, distance of flow propagation and duration time of rainfall

Let us discuss the relationship between the accuracy of unit hydrograph method, L_0 and t_d from the variations of α_t in the spatial and temporal scales. As explained above, the characteristics of runoff mechanism are divided into the two cases of (a) and (b). **Fig. 4.4** summarizes the relationship between them in which L_{01} and L_{03} express the flow routing distances belonging to the respective cases of (a) and (b) and L_{02} is the critical distance. It is obvious that $L_{01}\leq L_{02}\leq L_{03}$.

For the case of (a), the regions of $\partial\alpha_t/\partial t\geq 0$ or $\partial\alpha_t/\partial t<0$, where α_t is affected by temporal distribution of rainfall, decreases as L_0 increases, and the region vanishes at the critical distance (c) between (a) and (b). For the case of (b), the region of $\partial\alpha_t/\partial t=0$, where α_t is not affected by temporal distribution of rainfall, increases as L_0 increases. However, because the accuracy of unit hydrograph method increases as the time variation of α_t decreases as explained earlier, the accuracy of unit

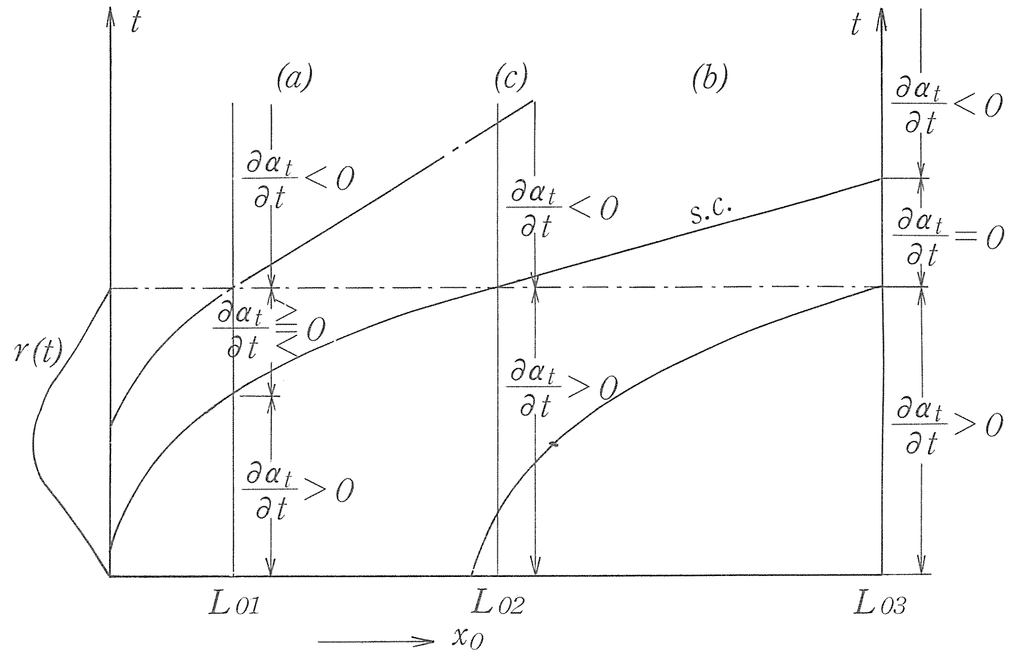

Fig. 4.4 Hourly variation of α_t on x_0~t plane in the combined case of (a) and (b)

hydrograph increases with an increase of L_0. However, this statement is correct only for the case where the temporal condition of rainfall t_d is independent of the spatial condition of catchment L_0. The above result is not valid, if there exists a high correlation between the two conditions.

The temporal scale of rainfall (i.e., duration time t_d) has an inverse relation with L_0 in terms of α_t. t_d should be examined with respect to rainfall amount which is the main cause of runoff phenomena, because it influences the magnitude of flood runoff. In the case where the amount of rainfall is proportional to rainfall intensity and the temporal distribution of rainfall is similar to each other, the decreasing effect of t_d on the accuracy of unit hydrograph method is equivalent to the increasing effect of L_0. The decrease of t_d, however, helps reduce the rainfall intensity as well as flood runoff but this issue would bring no practical problems.

In real situations where a large amount of rainfalls cause big floods, the decrease of t_d makes the average rainfall intensity increase and thus it leads to the increase of nonlinearity of phenomena. It is well known that the decrease of t_d makes the average rainfall intensity larger and its time variation more abrupt. The decrease of t_d tends to lower the accuracy of unit hydrograph method from an engineering viewpoint.

(4) Spatial distribution of rainfall and accuracy of unit hydrograph method

It is generally needed to separate the occurrence field from the propagation field, when dealing with a large scale phenomena. The former corresponds to the rainfall area in a catchment, while the latter a region from the rainfall area to the gauging station. Let L_0 be the length representing a scale of catchment area. L_0 can be expressed by the equation with reference to **Fig. 4.5** as follows:

$$L_0 = L_n + L_r + L_p$$

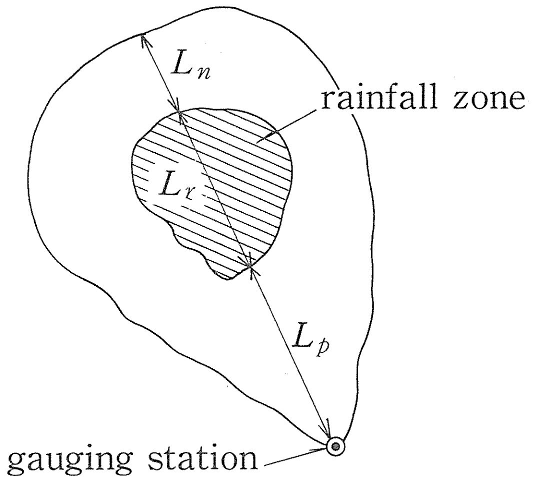

Fig. 4.5 Schematic diagram showing the relation between areal rainfall distribution and runoff route

where L_n is the distance (area) which does not contribute to runoff, L_r is the distance (area) with rainfall event, and L_p is the distance (area) transmitting runoff phenomena. Therefore, the distances to directly affect runoff are L_r and L_p and the spatial distribution of rainfall in these areas should be taken into consideration. Let us consider the relationship between the sizes of L_p and L_r and α_t. For the case where L_p is not equal to zero, the portion of $\partial\alpha_t/\partial t>0$ decreases at the downstream end so that the portion of $\partial\alpha_t/\partial t=0$ increase as L_p increases. As shown in **Fig. 4.6**, the increase of L_p makes the time variation of α_t decrease and consequently the accuracy of unit hydrograph method increases.

If L_p increases, rainfall losses such as infiltration and others also increase and an average effective rainfall decreases. As a result, the accuracy of unit hydrograph method becomes much higher. The above observation discloses that as the distance increases from the rainfall area producing dominant flood runoff to the gauging station, the applicability of unit hydrograph method increases. In other words, the accuracy of unit hydrograph method increases with an increase of L_p/L_r. In general, the increase of L_p implies an increase of catchment area contributing to runoff and gives rise to the uniform time distribution of rainfall. Therefore, the increase of L_o might lead to the higher accuracy of unit hydrograph method.

Furthermore, the average rainfall intensity decreases with an increase of catchment area. This fact also implies that the linearity of runoff phenomena is enhanced by the increase of catchment area, which tends to increase the accuracy of unit hydrograph method.

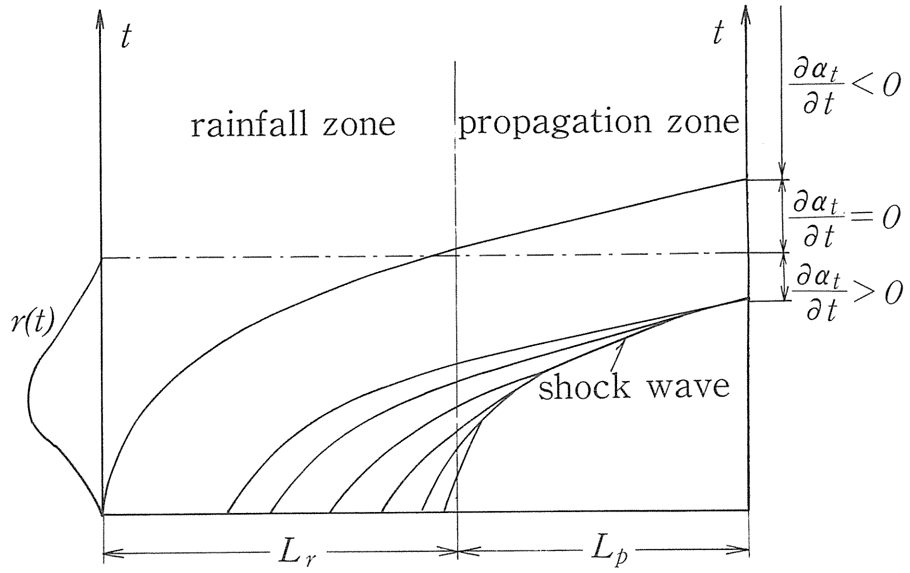

Fig. 4.6 Hourly variation of α_t in the case of $L_p \neq 0$

The unit hydrograph method is based on the assumption that runoff hydrograph is produced by spatially uniform rainfall in a catchment. In the case where the rainfall is nonuniformly distributed over a large catchment, the basic assumption cannot be supported, even though the effects of temporal variation of α_t and the average rainfall intensity become much smaller. Such a situation makes the accuracy of unit hydrograph method lower. It is said that the application of unit hydrograph method is not appropriate in a catchment with an area larger than 2,000 mile^2 in the United States of America. This fact would be because of the nonuniformity of spatial rainfall distribution.

(5) Summary on the accuracy of unit hydrograph method

It is clear from the above discussions that there are intercorrelations between the variation of α_t caused by the nonlinearity of the momentum equation and characteristics of rainfall and catchment, and the accuracy of unit hydrograph method tends to decrease as the variance of $\alpha_t = \{\int_\tau^t r(t)dt\}^{1/p-1}$ becomes large. Assuming that rainfall $r(t)$ is a random function of time t only, it is understood that the set of α_t is regarded as a sum of $r(t)$ within duration time $(t-\tau)$ restricted by the physical condition (i.e., a sum of subsets of set $r(t)$). It has been proved statistically that the mean of those sets tends to approach a constant value with an increase of duration time $(t-\tau)$. Therefore, the physical condition yielding the increase of time duration leads to the higher accuracy of unit hydrograph method.

The characteristics relating to the linearity of phenomena such as average rainfall intensity, catchment roughness and gradient of slope has the same meanings as the characteristics of flow distance and the rainfall duration, which are both connected with the accuracy or applicability of unit hydrograph method and the temporal scale of duration time $(t-\tau)$ in the statistical set $r(t)$. However, it should be kept in mind that there is an entire difference in the physical meanings of those two characteristics. The necessary condition on the relationship between the variation of α_t and the accuracy of unit hydrograph method is limited only in the vicinity of peak discharge. There is no explicit relationship between the variation of α_t, which varies from 0 to a certain value, and the size of catchment for a whole runoff process. However, if interests are just centered on the behavior near peak discharge, there is some correlation between the size of catchment and the characteristics relating to the linearity, even though the variance of α_t decreases with an increase of catchment size. Because the rainfall distribution changes both in time and space, it is instructive to investigate the relationship between the extent of field size and the accuracy of unit hydrograph method.

The relationship between the linearity and the accuracy of unit hydrograph method has to be uniquely and universally determined from a dynamic perspective. However, a unique relationship cannot be found between the elements of catchment scale and accuracy of unit hydrograph method, because of interwoven correlations among the elements of characteristics.

Based on the above discussions, the following cases are attributed to the increase of accuracy of unit hydrograph method in the light of linearity of runoff process:

i) The average rainfall intensity is small and variation of temporal rainfall distribution is rather smooth.

ii) The roughness of catchment is large and hence the condition of vegetation is an important factor for it.

iii) The average slope of catchment is small.The applicability of unit hydrograph method increases in the following cases, when viewed from the size and shape of a catchment;

iv) When a catchment area is too large, the rainfall tends to be nonuniformly distributed in space and there is a decline in the applicability of unit hydrograph method. For such a situation, it is preferable to divide an entire catchment into several subcatchments.

v) When the shape of catchment is of concentration type, the spatial distribution of rainfall tends to be uniform. This effect results in the elongation of flow routing distance as well as the synthetic distance of L_0 in a catchment.

vi) There might exist high correlations between a total amount, time distribution and duration of rainfall. It is anticipated that for the case where the rainfall duration is relatively long, the above conclusion would become opposite, if the rainfall is assumed to be a random function of time only.

5. CLASSIFICATION OF RUNOFF MECHANISMS AND APPLICABLE CONDITIONS OF MAXIMUM DISCHARGE THROUGH THE UNIT HYDROGRAPH METHOD

As the runoff mechanism is governed by the characteristics of rainfall and catchment, the unit hydrograph method must be treated according to the difference of runoff processes. Even if the units of intensity and time are suitably selected, the unit hydrograph cannot satisfy the hypothesis of unit hydrograph method for a whole runoff process in time and space, because α_t is a function of time. In a real situation, the temporal distribution of α_t can only be dealt with, because the runoff at the fixed point results from a lumping process in a single catchment. As α_t is generally a function of time, the whole runoff process in time at the gauging station, cannot exactly be represented by the unit hydrograph method. Therefore, when applying the unit hydrograph method to runoff problems, the unit rainfall or the elements of unit hydrograph will be changed by focusing on the particular parts of runoff mechanism. Although the unit rainfall is often regarded as the impulse rainfall, such an approach is based on the premise of the linearity of unit hydrograph method. This idea accrues from ignorance of the actual runoff process.

This chapter is mainly concerned with the classification of runoff processes according to the difference of application procedures of unit hydrograph method, which is followed by the investigation of the applicable conditions of unit hydrograph method. Of particular interest herein is the peak discharge in each classified runoff process from an engineering viewpoint.

(1) Averaging of α_t

When it is intended to estimate the peak discharge by the use of unit hydrograph method, given hyetograph, the most important parameter is α_{tp}, which is the value of α_t on the characteristic curve which generates the maximum discharge (the characteristic curve which generates the maximum discharge is denoted by "m.c." hereafter). When the integral on "m.c." is represented as $\int_s$, the mean, $(\alpha_{tp})_m$ of α_{tp} is given by

$$(\alpha_{tp})_m = \int_s \alpha_{tp} dt \Big/ \int_s dt \tag{5.1}$$

Therefore, as "m.c." increases monotonously, eq.(5.1) is equal to the following integral:

$$(\alpha_{tp})_m = \int_{\tau p}^{tp} \alpha_{tp} dt \Big/ \int_{\tau p}^{tp} dt \tag{5.2}$$

where τ_p and t_p are the starting and arriving times on "m.c.", respectively.

When using eqs.(4.2) and (3.2), the following equation is derived:

$$\int_{\tau_p}^{t_p} \alpha_{tp} dt = pL_0 K_0^{1/p}$$

and thus eq.(5.2) reduces to

$$(\alpha_{tp})_m = pL_0 K_0^{1/p} / (t_p - \tau_p) \quad \textbf{(5.3)}$$

The above relation is also deduced in such a way that the average tangent of "m.c." (i.e., average velocity of propagation) results from L_0 divided by propagation time $(t_p-\tau_p)$, which is multiplied by $pK_0^{1/p}$ to yield $(\alpha_{tp})_m$.

Since $(t_p-\tau_p)$ in eq.(5.3) is the propagation time of maximum discharge, the occurrence condition of peak discharge between t_p and τ_p is equal to the relation as follows:

$$r(\tau_p) = r(t_p)$$

Additionally, when multiple peaks are generated, depending on the rainfall conditions, the following equation must be satisfied:

$$t_p - \tau_p \leq t_{pi} - \tau_{pi}$$

where τ_{pi} and t_{pi} are starting and arriving times of the characteristic curve generating the peak discharge, respectively.

The quantity of $(t_p-\tau_p)$ differs, depending on whether t_p occurs during the rain or after it stops. In the case of $t_p<t_d$ (see (2), (a) in the previous chapter), the following expression is obtained, using eq.(2.8):

$$t_p - \tau_p = K_0 L_0^p / r_{mp}^{1-p} \quad \textbf{(5.4)}$$

where

$$r_{mp} = \int_{\tau_p}^{t_p} r(t) dt \bigg/ \int_{\tau_p}^{t_p} dt$$

On the other hand, in the case of $t_p>t_d$ (see (2), (b) in the previous chapter), the following equation is derived from eq.(2.24):

$$t_p - \tau_p = \left[pL_0 K_0^{1/p} + (1-p) r_m^{1/p-1} t_d^{1/p}\right] \Big/ (r_m t_d)^{1/p-1} \quad \textbf{(5.5)}$$

where

$$r_m = \int_0^{td} r(t) dt \bigg/ \int_0^{td} dt$$

Thus, from eq.(5.3) to eq.(5.5), the mean of α_{tp} is computed, which becomes a controlling factor of unit hydrograph method as described in the proceeding chapter. The mean of α_{tp} is summarized as follows:

$$t_p < t_d \; ; \quad (\alpha_{tp})_{m \circ s} = \left(pL_0 K_0^{1/p}\right)^{1-p} \left(p r_{mp}^{1/p-1}\right)^p \quad \textbf{(5.6)}$$

$$t_p > t_d \ ; \quad (\alpha_{tp})_{m \cdot l} = \{pL_0 K_0^{1/p} (r_m t_d)^{1/p-1}\} / \{pL_0 K_0^{1/p} + (1-p) r_m^{1/p-1} t_d^{1/p}\} \tag{5.7}$$

(2) Classification based on the difference of runoff mechanisms

Let t_u, t_d and t_c be the time unit, rainfall duration and time of concentration, respectively. The following three cases are considered according to the difference of runoff mechanism.

(a) The case of $t_u<t_c$ and $t_d>t_c$: This is the case where the gradient of catchment is steep, the roughness is small and the runoff routing distance is small. The propagation from the top of slope does not arrive at the downstream end during the duration time of unit rainfall, and the rainfall duration time is longer than the time of concentration. These situations almost coincide with the cases where the unit hydrograph method can be applied in Japan, and this is termed as "the case of a small catchment".

(b) The case of $t_u<t_c$ and $t_d<t_c$: Both unit time and rainfall duration are shorter than the time of concentration, and the flow routing distance is large. The maximum discharge appears to occur after the rain stops. Such cases are usually discerned in the rivers in the United States of America[13]. It is termed as the "case of a large catchment".

(c) The case of $t_u>t_c$ and $t_d>t_c$, and $t_u>t_c$ and $t_d<t_c$: The unit time is assumed to be longer than the time of concentration. This approach is not appropriate from a viewpoint that the unit time should be chosen to reduce the calculation error of unit hydrograph method to the least. Therefore, the two cases of (a) and (b) will be adopted in the unit hydrograph method.

(3) Applicable conditions of runoff analysis by unit hydrograph method

Let us deduce the necessary conditions to obtain the maximum discharge by use of unit hydrograph method for the above three cases. $(\alpha_{tp})_{m \cdot u}$ is defined as $(\alpha_{tp})_m$ of unit rainfall, while $(\alpha_{tp})_{m \cdot n}$ that of given hyetograph.

(a) The case of $t_u<t_c$ and $t_d>t_c$: When the maximum discharge is represented as $Q_{n \cdot p}$, the following equation is obtained:

$$Q_{n \cdot p} = \{(\alpha_{qp})_{m \cdot n} / K_0^{1/p}\} \int_{\tau_p}^{t_p} r(t) dt \tag{5.8}$$

The condition of $(\alpha_{tp})_{n \cdot u} \leq (\alpha_{tp})_{m \cdot n}$ indicates that the average propagation velocity which generates the maximum discharge for given hyetograph is equal to or larger than the average propagation velocity of the unit rainfall. In addition, the peak of unit hydrograph is assumed to continue from t_u to the arriving time of propagation. When using the above assumption and the two hypotheses of linearity on unit hydrograph method, the maximum discharge $Q_{u \cdot p}$ of unit hydrograph method is found to be

$$Q_{u\circ p}=\frac{(\alpha_{qp})_{m\cdot u}}{K_0^{1/p}}\left\{m_1\int_{\tau_p}^{t_{ui=1}}r_u dt+\sum_{i=2}^{t} m_i\int_0^{t_u}r_u dt+m_{i+1}\int_{t_p-t_{ui=i}}^{t_p}r_u dt\right\} \tag{5.9}$$

where r_u is the intensity of unit rainfall, $(\alpha_{qp})_m$ is the parameter for runoff corresponding to $(\alpha_{tp})_m$ whose relationship is given by eq.(4.5). The meaning of number i is such that the interval t_u including t_p is defined as i=1, while interval t_u including t_p is i+1 as shown in **Fig. 5.1**. The impact of unit rainfall before i=1 on the runoff is small and can be negligible.

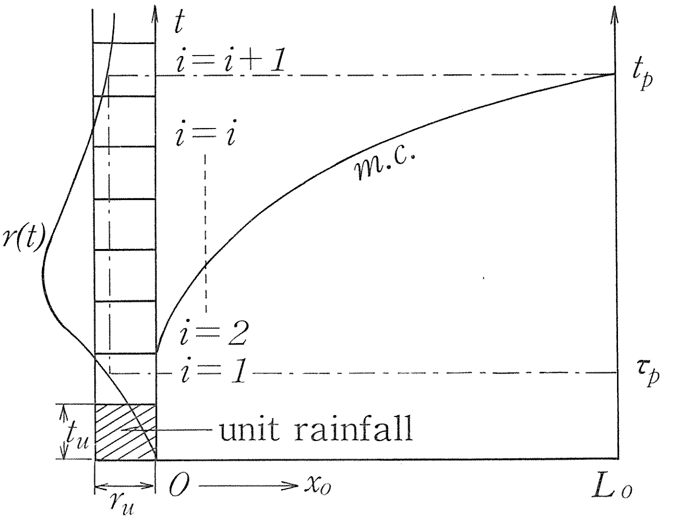

Fig. 5.1 Schematic diagram showing the meaning of number i in the case of t_u<t_c, t_d>t_c

The following equation is derived from the hypothesis of unit hydrograph method:

$$m_i=\left[\int_0^{t_u}r(t)dt\right]_i\Big/\int_0^{t_u}r_u dt$$

Then, eq.(5.9) is rewritten by

$$Q_{u\circ p}=\frac{(\alpha_{qp})_{m\circ u}}{K_0^{1/p}}\left\{\frac{\left[\int_0^{t_u}r(t)dt\right]_{i=1}}{\int_0^{t_u}r_u dt}\int_{\tau_p}^{t_{ui=1}}r_u dt+\sum_{i=2}^{i}\left[\int_0^{t_u}r(t)dt\right]_i+\frac{\left[\int_0^{t_u}r(t)dt\right]_{i=i+1}}{\int_0^{t_u}r_u dt}\int_{t_p-t_{ui=i}}^{t_p}r_u dt\right\} \tag{5.10}$$

Furthermore, eq.(5.8) is transformed as follows:

$$Q_{n\circ p}=\frac{(\alpha_{qp})_{m\circ n}}{K_0^{1/p}}\left\{\int_{\tau_p}^{t_{ui=1}}r(t)dt+\sum_{i=2}^{i}\left[\int_0^{t_u}r(t)dt\right]_i+\int_{t_p-t_{ui=i}}^{t_p}r(t)dt\right\} \tag{5.11}$$

When t_u is chosen to be adequately small, the following approximate equations are obtained:

$$\left.\begin{aligned}\int_{\tau_p}^{t_{ui=1}} r(t)dt &\fallingdotseq \left\{\left[\int_0^{t_u} r(t)dt\right]_{i=1} \Big/ \int_0^{t_u} r_u dt\right\} \int_{\tau_p}^{t_{ui=1}} r_u dr \\ \int_{t_p-t_{ui=1}}^{t_p} r_{(t)}dt &\fallingdotseq \left\{\left[\int_0^{t_u} r(t)dt\right]_{i=i+1} \Big/ \int_0^{t_u} r_u dt\right\} \int_{t_p-t_{ui=i}}^{t_p} r_u dt\end{aligned}\right\}$$

Note that both values of brackets { } in $Q_{n\cdot p}$ and $Q_{u\cdot p}$ are identical. Thus, when $(\alpha_{qp})_{m\cdot n}=(\alpha_{qp})_{m\cdot u}$, $Q_{n\cdot p}\fallingdotseq Q_{u\cdot p}$. The following condition is expected to be satisfied in terms of the relation of eq.(4.5) between α_q and α_t:

$$(\alpha_{tp})_{m\circ n}=(\alpha_{tp})_{m\circ u} \tag{5.12}$$

The maximum discharge for a given hyetograph coincides with the result calculated by the unit hydrograph method in both time distribution and volume under the condition of eq.(5.12).

(b) The case of $t_u<t_c$ and $t_d<t_c$: Because of $\tau_p=0$, $Q_{n\cdot p}$ is formulated as follows:

$$\begin{aligned} Q_{n\circ p} &= \left\{(\alpha_{qp})_{m\circ n}\Big/K_0^{1/p}\right\}\int_0^{t_d} r(t)dt \\ &= \left\{(\alpha_{qp})_{m\circ n}\Big/K_0^{1/p}\right\}\left\{\sum_{i=1}^{i}\left[\int_0^{t_u} r(t)dt\right]_i\right\}\end{aligned} \tag{5.13}$$

Then, by giving the condition of $(\alpha_{tp})_{m\cdot u}\leq(\alpha_{tp})_{m\cdot n}$ and introducing the concept of (a), the following equation is obtained:

$$Q_{u\circ p}=\left\{(\alpha_{qp})_{m\circ u}\Big/K_0^{1/p}\right\}\left\{\sum_{i=1}^{i} m_i\int_0^{t_u} r_u dt\right\} \tag{5.14}$$

The hypothesis of the unit hydrograph method gives

$$m_i=\left[\int_0^{t_u} r(t)dt\right]_i \Big/ \int_0^{t_u} r_u dt$$

Thus, $Q_{u\cdot p}$ is given by

$$Q_{u\circ p}=\left\{(\alpha_{qp})_{m\circ u}\Big/K_0^{1/p}\right\}\left\{\sum_{i=1}^{i}\left[\int_0^{t_u} r(t)dt\right]_i\right\} \tag{5.15}$$

where the number i indicates that the first t_u section is represented by i=1 and other sections are sequentially numbered as 2, 3, ..., i as shown in **Fig. 5.2**.

From eqs.(5.13) and (5.15), it is obvious that when $(\alpha_{qp})_{m\cdot n}$ is equal to $(\alpha_{qp})_{m\cdot u}$, the following relation can be satisfied:

$$(\alpha_{tp})_{m\circ n}=(\alpha_{tp})_{m\circ u} \tag{5.16}$$

$Q_{n\cdot p}$ is equal to $Q_{u\cdot p}$ under the condition of eq.(5.16).

(c) The case of $t_u>t_c$ and $t_d>t_c$, and $t_u>t_c$ and $t_d<t_c$: The following equations are formulated:

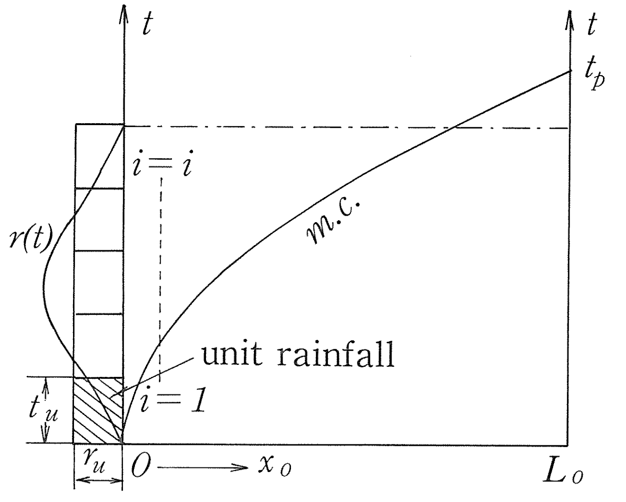

Fig. 5.2 Schematic diagram showing the meaning of number *i* in the case of $t_u<t_c$, $t_d<t_c$

$$Q_{n \cdot p}=\left\{(\alpha_{qp})_{m \cdot n}/K_0^{1/p}\right\}\int_{\tau_p}^{t_p} r(t)dt \tag{5.17}$$

$$Q_{u \cdot p}=\left\{(\alpha_{qp})_{m \cdot u}/K_0^{1/p}\right\}m\int_{\tau_p}^{t_p} r_u dt \tag{5.18}$$

where

$$m=\int_0^{t_u} r(t)dt \Big/ \int_0^{t_u} r_u dt$$

If the relation of $\int_{\tau p}^{tp} r(t)dt=\int_{\tau p}^{tp} r_u dt$ would be valid, the following approximate relationship can be satisfied:

$$\int_{\tau_p}^{t_p} r(t)dt \fallingdotseq \left\{\int_0^{t_u} r(t)dt \Big/ \int_0^{t_u} r_u dt\right\}\int_{\tau_p}^{t_p} r_u dt$$

However, such a premise violates the basic concept of unit hydrograph method, because this method does not allow the rainfall to be highly variable over a catchment.

Therefore, the subsequent discussions will be limited to the two cases of (a) small catchment and (b) large catchment by using applicable conditions for maximum discharge of $(\alpha_{tp})_{m \cdot u}=(\alpha_{tp})_{m \cdot n}$ on the unit hydrograph method.

6. ELEMENTS OF THE UNIT RAINFALL

The present chapter is intended to clarify what kind of relationship ought to exist between the unit rainfall, and rainfall and catchment characteristics to correctly estimate the peak of hydrograph through unit hydrograph theory by using the applicable condition of $(a_{tp})_{mu}=(a_{tp})_{mn}$ obtained in the previous chapter.

(1) The case of a small catchment ($t_u<t_c,\ t_d>t_c$)

The following relations can hold in terms of unit rainfall and runoff mechanisms with actual rainfall:

$$\left.\begin{aligned} &B_c=pL_0K_0^{1/p},\ \delta=(1/p)-1,\ \varepsilon=1-p, \\ &r_{mp}=\int_{\tau_p}^{t_p}r(t)dt \Big/ \int_{\tau_p}^{t_p}dt \end{aligned}\right\}$$

The use of the above equations coupled with eqs.(5.6) and (5.7) yields

$$(\alpha_{tp})_{m\cdot u}=B_c(r_ut_u)^{\delta}\Big/\left\{B_c+\varepsilon r_u^{\delta}t_u^{\delta+1}\right\} \tag{6.1}$$

$$(\alpha_{tp})_{m\cdot n}=p^{p}(B_cr_{mp})^{\varepsilon} \tag{6.2}$$

When using the applicable condition described in (3) of Chapter 5, the relation is obtained by setting the above two equations to be identical as follows:

$$r_u^{\delta}\left\{t_u^{\delta}/p^{p}(B_cr^{mp})^{\varepsilon}-\varepsilon t_u^{\delta+1}/B_c\right\}=1 \tag{6.3}$$

Because of $r_u^{\delta}>0$, the following inequality can hold:

$$t_u<B_c^{\varepsilon-1}\Big/(\varepsilon p^{p}r_{mp}^{\varepsilon}) \tag{6.4}$$

The unit time t_u, which was empirically selected in the past satisfies the above inequality sufficiently. Transformation of eq.(6.3) gives

$$r_u=\left\{(p^{p}B_cr_{mp}^{\varepsilon})^{1/\delta}/(B_c^{p}-\varepsilon p^{p}r_{mp}^{\varepsilon}t_u)^{1/\delta}\right\}/t_u \tag{6.5}$$

In general, $B_c^p \gg \varepsilon p^p r_{mp}^{\varepsilon} t_u$ and hence eq.(6.5) can be approximated by

$$r_u=\left(p^{1/\delta}B_c r_{mp}\right)^p/t_u \quad (6.6)$$

The use of Manning's resistance law of p=0.6, ϵ=0.4, and δ=2/3, reduces the above expression to

$$r_u=0.6^{0.9}(B_c r_{mp})^{0.6}/t_u \quad (6.7)$$

It is of interest to note that the intensity of unit rainfall, r_u is proportional to the product of catchment index, B_c and average rainfall intensity, r_{mp} with the power of 0.6 as well as inversely proportional to the unit time t_u. Substitution of $B_c=pL_0K_0^{1/p}$ into eq.(6.7) yields

$$r_u=0.6^{1.5}(n_0L_0r_{mp}/\sqrt{\sin\theta_0}\ )^{0.6}/t_u \quad (6.8)$$

It follows that the total amount of unit rainfall, r_ut_u should be proportional to r_{mp} with the power of 0.6 in a catchment.

As a typical example, the following values are assumed:

$$t_u=2\,hr,\quad r_{mp}=10\ mm/hr=10^{-2}m/hr,\quad L_0=1000\ m,^*$$
$$n_0=1\,m^{-1/3}\cdot s$$
$$=1/3600\ m^{-1/3}hr^{**},\quad \sin\theta_0=1/400$$

As a result, r_u can be calculated as r_u=0.05 m/hr=50 mm/hr. It is emphasized herein that this value is obtained from hydraulic considerations of the rainfall-runoff process. The present approach is significantly different from the conventional one where the value of r_u was often calculated from the observed hydrograph by using the linear assumption of unit hydrograph method. It is natural that the intensity of unit rainfall becomes extremely large with the method described in the present section, because the rainfall-runoff process is assumed to be essentially nonlinear. The

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* L_0 is not large in hydrologic practices, because this quantity is mainly dominated by a flowing distance on the slope.

** n_0 is mainly determined by the roughness coefficient on the slope n'. For instance, n' =1~1.5 $m^{-1/3}\cdot s$ in the Yura River basin and roughly 0.3 $m^{-1/3}\cdot s$ in the Odo River basin. Palmer also reports the value of n'=0.2~0.4 $m^{-1/3}\cdot s$ in the grass-covered experiments.

conventional concept of unit rainfall seldom has physical significance in a sense that it only makes the total volume given by the unit hydrograph coincide with that of unit rainfall. Arbitrary values such as 1 mm or 1 inch per unit area were chosen only for convenience of computation. Therefore, it is easily understood from the above argument that no relationship exists between the unit rainfall and unit hydrograph from the viewpoint of runoff mechanisms. Furthermore, it is impossible to decide the elements of unit rainfall, especially, its unit intensity. However, it should be kept in mind that from a phenomenological or practical point of view, the unit time should be selected so as to express the whole hydrograph by using the method of unit hydrograph as accurately as possible. Such a premise is based on the fact that the unit time has been traditionally considered to be an important element in terms of unit rainfall.

Equation (6.5) or (6.6) ia a fundamental expression in terms of r_u and t_u, respectively, which is obtained from a hydraulic rationale. Both quantities cannot be determined independently. There exists only such a relationship between r_u and t_u so as to make the peak discharge coincide. The value of t_u can be identified in a statistical way based on error theory that the whole hydrograph has to be represented as adequately as possible. This is the main reason why t_u has traditionally been connected with the delay time of peak discharge.

The propagation time of peak discharge, t_{pc} is given from eq.(5.4) as

$$t_{pc}=B_c^{p}\Big/\left(p^{p} r_{mp}^{\varepsilon}\right) \tag{6.9}$$

When using the relation of eq.(6.6), t_u is expressed as

$$t_u=p^{1/\delta}\left(r_{mp}/r_u\right)t_{pc} \tag{6.10}$$

Note that the unit time is proportional to the propagation time of peak discharge as well as its delay time. It is obvious that there is a close relationship between the delay and propagation times of peak discharge. When the hyetograph is symmetrical around the peak rainfall, the delay time is equal to a half of the propagation time. Hence, the traditional approach is rational in the sense that t_u has been connected to the delay time of peak discharge.

Rewriting eq.(6.10) with p=0.6 gives

$$t_u=0.4(r_{mp}/r_u)t_{pc} \tag{6.11}$$

Under the conditions that $(\alpha_{tp})_{mu}=(\alpha_{tp})_{mn}$ and $t_u<t_c$, $t_d>t_c$, the relation of $r_u>r_{mp}$ always holds. When r_u is selected as twice the value of r_{mp}, the unit time should be 1/5 of the propagation time of peak discharge and is about 2/5 of its delay time.

(2) The case of a large catchment $(t_u<t_c,\ t_d<t_c)$

In this case, $(\alpha_{tp})_{mu}$ is given by eq.(6.1), while $(\alpha_{tp})_{mn}$ is expressed via use of eq.(5.7) as

$$(\alpha_{tp})_{m\circ n}=B_c(r_m t_d)^{\delta}\Big/\left\{B_c+\varepsilon r_m^{\delta} t_d^{\delta+1}\right\} \tag{6.12}$$

where

$$r_m=\int_0^{t_d} r(t)dt\Big/\int_0^{t_d}dt$$

Under the assumption of $(\alpha_{tp})_{mu}=(\alpha_{tp})_{mn}$, the following relationship exists between r_u and t_u, which satisfies the hypothesis of unit hydrograph method in terms of peak discharge:

$$r_u t_u/r_m t_d=B_c^{1/\delta}\Big/\left\{B_c+\varepsilon(r_m t_d)^{\delta}(t_d-t_u)\right\}^{1/\delta} \tag{6.13}$$

In general, the relation of $B_c \gg \varepsilon(r_m t_d)^{\delta}(t_d-t_u)$ can hold and hence the above equation is approximated by

$$r_u t_u=r_m t_d=R_{td} \tag{6.14}$$

It is clear that the product of r_u and t_u is equal to the total rainfall amount R_{td} and the relationship between r_u and t_u is inversely proportional to each other for the fixed value of R_{td}. As discussed in the preceding section, the physical significance of r_u and t_u for a large catchment is the same as for a small catchment. The value of t_u should be chosen to reproduce the whole hydrograph with high accuracy. In the United States, t_u is often selected to be 1/2~1/4 of the delay time of peak discharge. Such an idea might be recognized as one of the criteria for the choice of t_u.

(3) Relationship between catchment scale and elements of unit rainfall

Let the subscript of s and l be the element of unit rainfall in small and large catchments, respectively. The relations are obtained from eqs.(6.6) and (6.14) as follows:

$$r_{u\circ s}=p^{p/\delta}(B_c r_{m\circ s})^{p}/t_{u\circ s}\ ,\quad r_{m\circ s}=\int_{\tau_p}^{t_p} r(t)dt\Big/\int_{\tau_p}^{t_p}dt \tag{6.15}$$

$$r_{u\circ l}=r_{m\circ l}t_d/t_{u\circ l}=R_{td}/t_{u\circ l}\ ,\quad r_{m\circ l}=\int_0^{t_d} r(t)dt\Big/\int_0^{t_d}dt \tag{6.16}$$

In the light of the above two relations, careful considerations should be laid on the choices of unit rainfall for each of small and large catchments, which are summarized below:

i) The catchmen index, B_c is an important factor in a small catchment where the elements of unit rainfall should be changed. On the other hand, it is not necessary to consider such a catchment characteristics in a large catchment.

ii) The rainfall intensity in the vicinity of peak rainfall is important for a small catchment, while the average intensity of hyetograph and rainfall duration or total rainfall amount are of practical importance for a large catchment.

iii) The above two discussions show that the catchment characteristics as well as the intensity in the vicinity of peak rainfall are relevant factors in Japanese rivers where the rainfall duration is of less importance, even though the elements of unit rainfall should be changed with the variation of rainfall intensity for a small catchment. It is well recognized that the total rainfall amount and its duration are much more important for large river basins in the United States where catchment characteristics is not an influential factor.

iv) The value of $r_{m\cdot s}$ is defined as the intensity near the peak rainfall. The relation of $r_u > r_{m\cdot s}$ can be satisfied, because of $p^{1/\delta} t_{pc} > t_u$ in eq.(6.10) which is equivalent to eq.(6.15). The value of $r_{m\cdot l}$ is the average rainfall intensity of hyetograph and hence a rather larger value should be adopted for the unit intensity in a small catchment than for that in a large one where the total rainfall amount of unit rainfall should be taken as large as that of actual rainfall. This statement is qualitatively supported by the fact that the unit rainfall intensity of 10~20 mm/hr is usually adopted in Japan, while 1 inch/day in the United States.

v) The delay time of peak discharge has been traditionally taken to be a representative index of unit time. It is reported in the United States [13] that the rainfall duration is 1~2 times the delay time of peak discharge. Thus, even if the unit time is set as 1/2~1/4 of its delay time, the unit hydrograph method is easily implemented. When the same procedure is simply applied to runoff analyses in Japan, the computation becomes rather cumbersome. As discussed in the subsequent chapters, quantitative criteria are needed to determine the time unit in Japan, because the hydraulic significance with respect to the delay time of peak discharge is different between Japan and the United States. It would not make sense to simply follow the way used in the United States without taking into consideration actual hydrologic phenomena and theoretical backgrounds.

7. ELEMENTS OF THE UNIT HYDROGRAPH

The important elements of unit hydrograph are the peak, time length of its rising limb, and the base length. Among them, the peak and time length of the rising stage directly affect the accuracy of unit hydrograph. So far many methods have been proposed to determine these elements. However, there are some problems to be solved yet because the methods depend only on empirical facts and lack theoretical backgrounds. In this chapter, the peak and the rising time of unit hydrograph, which make the maximum discharge and its occurrence time coincide, are determined from a hydraulic viewpoint based on the results obtained from the previous chapter.

(1) Delay time of maximum discharge

The present study has already emphasized the fact that the runoff mechanisms are dominated by the propagation state of rainfall. On the other hand, the conventional method is based on the idea that the time delay of maximum discharge, an expression of the propagation state of rainfall, can be related with the time length of the rising limb of unit hydrograph.

The delay time of maximum discharge is equivalent to the time required for the occurrence of maximum discharge from a certain reference point of time with respect to rainfall, and is recognized as the synthetic effect of all the rainfall and basin characteristics. It is measured by the time required for the rainfall to move down to the gauging station from the gravity center of catchment. Such a concept is unclear. As for the reference time of rainfall, Snyder[10] defines the gravity center of hyetograph, which is widely used in the United States, while the time of maximum intensity of rainfall is often used in Japan. These methods are based only on the empirical facts and lack the theoretical background. Therefore, let us herein consider the delay time of maximum discharge from a hydraulic standpoint.

It is found from eq.(2.38) that there is a relation of $r(\tau_p)=r(t_p)$ between the starting time τ_p and arriving time t_p of propagation generating the maximum discharge and hence the rainfall within the time $(t_p-\tau_p)$ contributes to the maximum discharge. Therefore, $(t_p-\tau_p)$ can be considered as the propagation time of rainfall with a direct connection to the maximum discharge, which is represented by t_{pc} hereafter. The time, t_{pc}, differs between the case of $t_{pc}<t_d$, where maximum discharge occurs within a rainfall duration, and the case of $t_{pc}>t_d$, where it occurs after the rain stops. Let $t_{pc\cdot s}$ and $t_{pc\cdot l}$ be the propagation times in the two cases, respectively. The following equations can be derived from eqs.(5.4) and (5.5):

$$t_{pc \cdot s} = B_c^p / (p\, {}^p r_{mp}^{\varepsilon}) \tag{7.1}$$

$$t_{pc \cdot l} = B_c / R_{td}^{\delta} + \varepsilon\, t_d \tag{7.2}$$

It follows from the results obtained in 5.(2) that the case of eq.(7.1) corresponds to a small catchment, while that of eq.(7.2) to a large one. Both of them are the propagation time of rainfall to be directly related to the maximum discharge and must be highly correlated to the delay time of maximum discharge. Let us define the reference time in terms of rainfall for delay time of maximum discharge to quantify this correlation.

As the propagation time is generally related to the size of catchment, the contributing portion of rainfall to the maximum discharge must decrease in a smaller catchment, which leads to the following criteria: in small catchments, the time of maximum rainfall intensity should be taken as the reference time in regard to rainfall, because the rainfall around the time of maximum discharge contributes to the maximum discharge as discerned from eq.(7.1). It is inadequate to take the gravity center of hyetograph as the reference time, especially in the case of highly asymmetrical rainfall distrbution. In a large catchment, on the other hand, there are many rainfall peaks during the long propagation time, and the time of peak rainfall cannot easily be fixed. Therefore, it is preferable to take the time of the gravity center of hyetograph as reference time, which does not reveal much variation in actual situations.

In a larger catchment where eq.(7.2) holds, it is better to take a starting time of the rainfall as the reference from a dynamical viewpoint, because the whole hyetograph contributes to the maximum discharge. The use of this reference time, however, causes much variation in the delay time of maximum discharge depending on the total amount and the duration of rainfall. Therefore, the time of gravity center of hyetograph should be taken from the viewpoint that the delay time of maximum discharge as an important runoff characteristics in a catchment should not be much up to rainfall conditions. This choice of reference time can reduce the influence of rainfall conditions on the delay time of maximum discharge.

In short, it is impossible to select a unique reference time because of the randomness of rainfall conditions, and a suitable type should be chosen according to the basin characteristics and its size. It is empirically known that the time of gravity center of hyetograph is used in the United States, while the time of maximum rainfall intensity in Japan. These circumstances result from the above discussions and it should be kept in mind that they have different significances.

Next, let us consider the relationship between the delay time of maximum discharge T_g and the propagation time t_{pc} where $T_{g \cdot s}$ and $T_{g \cdot l}$ vary with runoff mechanisms. The subscripts s and l denote the same meanings as in the case of t_{pc}. When the maximum discharge occurs during the rainfall duration, there exists the following relation:

$$T_{g \cdot s}=(1-\alpha)t_{pc \cdot s} \tag{7.3}$$

When taking the time of maximum rainfall intensity as the reference as shown in **Fig.7.1**, α is a parameter defined as the ratio of the difference from the starting time of propagation generating peak discharge, τ_p to the time of maximum rainfall intensity to the propagation time $t_{pc \cdot s}$. The parameter α is dominated by the shape of rainfall distribution during $\tau_{g \cdot s}$, which generally ranges from 0 to 1 and becomes 1/2 when the rainfall shape around its peak is symmetrical.

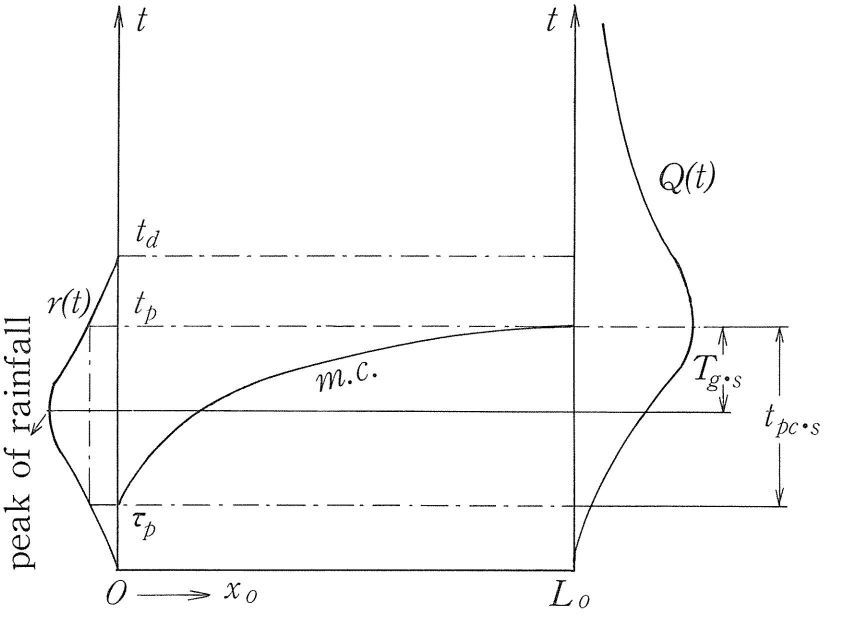

Fig.7.1 Propagation states of rainfall disturbance in the case when the peak flow discharge occurs before the end of rainfall duration

In the case where the maximum discharge occurs after the rain stops, the peak of hydrograph continues from t_d to $t_{pc \cdot l}$ if the flowing distance on the slope is the same at all the points. However, the maximum discharge occurs at some time in practice because natural basins have various shapes and flowing paths of rainfall are quite random. The time of maximum discharge might exist between t_d and $t_{pc \cdot l}$, because B_c or L_o is adopted to represent most effective basin characteristics in the runoff process. The broken line shown in **Fig.7.2** demonstrates the hydrograph along a single flowing path x_0 without considering the randomness of basin shape. An actual hydrograph can be shown by the solid line in the figure. Taking these features into account, $T_{p\cdot l}$ can be expressed as

$$T_{p\cdot l}=(1-\beta)t_{pc\cdot l}+\beta t_d \tag{7.4}$$

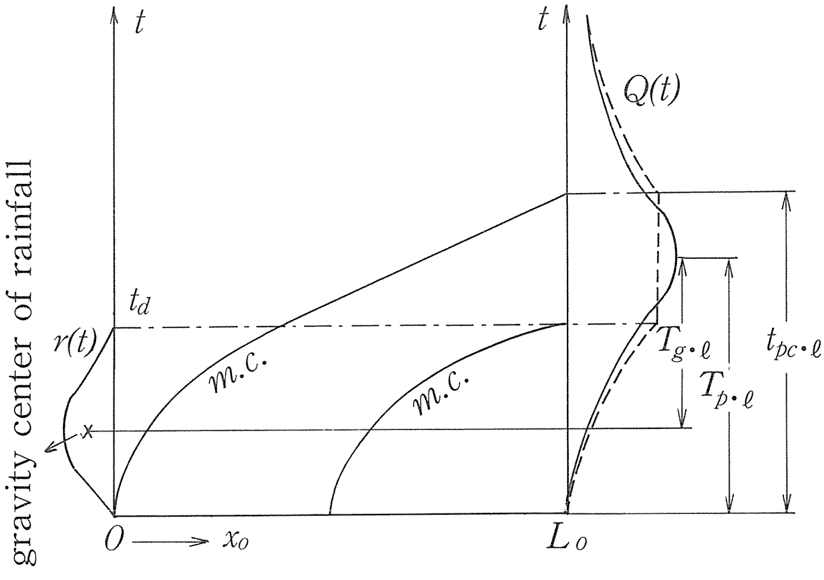

Fig.7.2 Propagation states of rainfall disturbance in the case when the peak flow discharge occurs after the end of rainfall duration

where β is the parameter which locates $T_{p\cdot l}$ between t_d and $t_{pc\cdot l}$ according to the randomness of basin shape and generally ranges from 0 to 1 with the mean value of 1/2.

Moreover, the delay time of maximum discharge, $T_{g\cdot l}$, is related to $T_{p\cdot l}$ from the following equation:

$$T_{g\cdot l}=T_{p\cdot l}-\beta' t_d \tag{7.5}$$

When taking the time of gravity center of rainfall distribution as the reference shown in **Fig.7.2**, β' is the parameter denoting the ratio of the time of gravity center to t_d (i.e., gravity center of hyetograph). This parameter generally ranges from 0 to 1 and is 1/2 when the hyetograpg reveals a symmetrical shape.

The relationship between $T_{g\cdot l}$ and $t_{pc\cdot l}$ can easily be obtained from eliminating $T_{p\cdot l}$ from eqs.(7.4) and (7.5) as follows:

$$T_{g\cdot l}=(1-\beta)t_{pc\cdot l}+(\beta-\beta')t_d \tag{7.6}$$

Rewriting $T_{g\cdot s}$ and $T_{g\cdot l}$ by using eqs.(7.1) and (7.2) yields

$$T_{g\cdot s}=(1-\alpha)B_c^p/(p^p r_{mp}^{\varepsilon}) \tag{7.7}$$

$$T_{g\cdot l}=(1-\beta)B_c/R_{td}^{\delta}+\{\varepsilon(1-\beta)+(\beta-\beta')\}t_d \tag{7.8}$$

where the values α, β and β' can be set as 1/2 on average. The above two equations can explain the empirical facts that T_g varies according to the rainfall intensity and shape around its peak in Japan (B_c=const.) and the report by Taylor and Schwarz[13] in that T_g is mainly dominated by rainfall duration and $T_{g\cdot l}$ increases as the duration becomes long in United States.

(2) Elements of unit hydrograph in a small catchment

The following discussion can be applied in small catchments as in Japanese rivers where the case of $t_u<t_c$ and $t_d>t_c$ are commonly valid. As the runoff mechanism produced by a unit rainfall, namely, the unit hydrograph, corresponds to the case discussed in (a) of 5.(2), the propagation time $t_{uc\cdot s}$ is expressed from eq.(2.18) as

$$t_{uc\cdot s}=B_c/(r_u t_u)^{\delta}+\varepsilon t_u \tag{7.9}$$

Because the unit hydrograph defined herein has hydraulic meanings, the rising time of unit hydrograph $t_{g\cdot s}$ is generally expressed from the same consideration in the previous section as follows:

$$t_{g\cdot s}=(1-\alpha')t_{uc\cdot s}+\alpha' t_u \tag{7.10}$$

where α' is the ratio of the time from $\tau_{g\cdot s}$ to $t_{g\cdot s}$ to the one from t_u to $t_{uc\cdot s}$, which is dominated mainly by the basin shape. This parameter ranges from 0 to 1, and becomes 1/2 when $t_{g\cdot s}$ is in the midst of t_u and $t_{uc\cdot s}$.

In order to obtain such elements of unit hydrograph that can reproduce the maximum discharge with high precision, the following relation is derived from eliminating r_{mp} from eqs.(6.5) and (6.9):

$$(r_u t_u)^{\delta}=B_c/(t_{pc\cdot s}-\varepsilon t_u) \tag{7.11}$$

Substituting this relation into eq.(7.9) gives

$$t_{uc\cdot s}=t_{pc\cdot s} \tag{7.12}$$

The above equation is also derived from the applicable condition for maximum discharge of $(\alpha_{tp})_{m\cdot u}=(\alpha_{tp})_{m\cdot n}$. Therefore, the rising time of unit hydrograph $t_{g\cdot s}$ can be expressed from eq.(7.10) as follows:

$$t_{g\cdot s}=(1-\alpha')t_{pc\cdot s}+\alpha' t_u \tag{7.13}$$

The use of eq.(7.3) yields

$$t_{g\cdot s}=\{(1-\alpha')/(1-\alpha)\}T_{g\cdot s}+\alpha' t_u \tag{7.14}$$

Equation (7.13) or (7.14) is the controlling equation of the rising time of unit hydrograph for a small catchment, which is expressed in terms of propagation time or delay time of maximum discharge. When adopting the value of 0.5 for α and α' which ranges from 0 to 1, the following approximate equation is obtained:

$$t_{g\cdot s}=T_{g\cdot s}+0.5t_u \tag{7.15}$$

Tategami[12] uses the value of 0.8 as a coefficient in the second term of the right-hand side in eq.(7.15). The value of $t_{g\cdot s}$ given by eq.(7.15) can match the maximum discharge. However, a slightly larger value should be assigned to improve the average accuracy of unit hydrograph method for the total runoff processes as Nakayasu[11] recommends. The relation of $t_{g\cdot s}=T_{g\cdot s}+0.5t_u$ given by eq.(7.15) is theoretically supported from the standpoint that the delay time of unit hydrograph is equal to that of the actual rainfall.

The peak of unit hydrograph is expressed from eq.(3.3) as

$$q_{up\cdot s}=(1/K_0^{1/p})\left(\int_0^{t_u} r_u dt\right)^{1/p} \tag{7.16}$$

Substitution of $r_u=(p^{1/\delta}B_c r_{mp})^p/t_u$ obtained from the approximate equation of unit rainfall (6.6) into eq.(7.16) gives the peak of unit hydrograph which can match the maximum discharge by the actual rainfall as follows:

$$q_{up\cdot s}=p^{1/\varepsilon}L_0 r_{mp} \tag{7.17}$$

The maximum discharge by the actual rainfall is given by

$$Q_{np}=L_0 r_{mp} \tag{7.18}$$

The relationship between $q_{up\cdot s}$ and Q_{np} is expressed as

$$q_{up\cdot s}=p^{1/\varepsilon}Q_{np} \tag{7.19}$$

For the case of p=0.6, the following equation holds:

$$q_{up\cdot s}=0.6^{2.5}Q_{np}=0.28Q_{np} \quad \textbf{(7.20)}$$

Therefore, the relationship between the two hydraulic variables is dominated by the exponent p representing the relationship between depth and discharge. The peak of unit hydrograph can be approximated by about 28% of the maximum discharge by the actual rainfall. From eqs.(7.18) and (6.9), Q_{np} is expressed as

$$Q_{np}=L_0(B_c/p)^{1/\delta}/t_{pc\cdot s}^{1/\varepsilon} \quad \textbf{(7.21)}$$

The relationship between $q'_{up\cdot s}$, the peak of unit hydrograph per unit area and the characteristics of rainfall, catchment and runoff is given by

$$q'_{up\cdot s}=p^{1/\varepsilon}r_{mp}=p^{1/\varepsilon}(L_0 n_0/\sqrt{\sin\theta_0})^{1/\delta}/t_{pc\cdot s}^{1/\varepsilon} \quad \textbf{(7.22)}$$

Note that $q'_{up\cdot s}$ decreases in a larger catchment because r_{mp} decreases with an increase of L_o, and that $q'_{up\cdot s}$ increases with a decrease of $t_{pc\cdot s}$, which conforms to the empirical facts.

Equations (7.13), (7.14) and (7.19) are the fundamental expressions to be used when a unit hydrograph is constructed from the observed discharge data. Let us consider how the elements of unit hydrograph $t_{g\cdot s}$ and $q_{up\cdot s}$ vary with rainfall characteristics, especially rainfall intensity. The following expression is derived from eqs.(7.13) and (6.9) as

$$t_{g\cdot s}=(1-\alpha')B_c^{p}/(p^{p}r_{mp}^{\varepsilon})+\alpha' t_u \quad \textbf{(7.23)}$$

For a particular catchment, $t_{g\cdot s}$ decreases with the increase of rainfall intensity. Since the first term of the right-hand side in the above equation is quite larger than the second term, while $t_{g\cdot s}$ decreases with the increasing rainfall intensity for a catchment, the relationship between the rising times of unit hydrograph $t_{g\cdot si}$ and $t_{g\cdot sj}$ corresponding to the average rainfall intensities r_{mpi} and r_{mpj}, respectively, is approximately given by

$$\left(t_{g\cdot s_i}/t_{g\cdot s_j}\right)=\left(r_{mp_i}/r_{mp_j}\right)^{\varepsilon} \quad \textbf{(7.24)}$$

In the same manner, the relationship between the peaks of unit hydrograph $q_{up\cdot si}$ and $q_{up\cdot sj}$ is derived from (7.17) as

$$\left(q_{up\cdot s_i}/q_{up\cdot s_j}\right)=\left(r_{mp_i}/r_{mp_j}\right) \quad \textbf{(7.25)}$$

As ϵ is considered to be about 0.4, the rising time of unit hydrograph does not largely vary with rainfall intensity, while the peak is linearly proportional to rainfall intensity, which shows the significant effect.

The fact that remarkable changes of the elements of unit hydrograph with the variation of rainfall intensity, which is inversely computed from the actual observations at the Yura River, appears to be an obvious evidence. It is theoretically verified by the results[15] that different unit hydrographs should be used for different rainfall intensities even in the same catchment.

(3) Elements of unit hydrograph in a large catchment

The following discussion can be applied in large catchments as in the rivers of the United States where the consideration of $t_u<t_c$ and $t_d<t_c$ is sufficient. The propagation time of unit hydrograph is given from eq.(2.8) as

$$t_{uc\cdot l}=B_c/(r_u t_u)^{\delta}+\varepsilon t_u \tag{7.26}$$

In view of the randomness of catchment shape in the preceding section, the rising time of unit hydrograph is expressed as follows:

$$t_{g\cdot l}=(1-\beta'')t_{uc\cdot l}+\beta'' t_u \tag{7.27}$$

where β'' is the ratio of the time from $t_{g\cdot l}$ to $t_{uc\cdot l}$ to the one from t_u to $t_{uc\cdot l}$, which is dominated mainly by the shape of catchment. This value ranges from 0 to 1, and is 1/2 when $t_{g\cdot l}$ takes the middle value between t_u and $t_{uc\cdot l}$.

When substituting the equation of unit hyetograph (6.13) and the propagation equation of maximum discharge into eq. (7.26), the following relation is obtained:

$$t_{uc\cdot l}=t_{pc\cdot l} \tag{7.28}$$

which can also be derived from the applicable condition of maximum discharge $(\alpha_{tp})_{m\cdot u}=(\alpha_{tp})_{m\cdot n}$. Therefore, the rising time of unit hydrograph is expressed from eq.(7.27) as follows:

$$t_{g\cdot l}=(1-\beta'')t_{pc\cdot l}+\beta'' t_u \tag{7.29}$$

The use of eq.(7.6) yields

$$t_{g\cdot l}=\frac{(1-\beta'')}{(1-\beta)}T_{g\cdot l}+\beta'' t_u-(1-\beta'')\frac{\beta-\beta'}{1-\beta}t_d \tag{7.30}$$

Equation (7.29) or (7.30) is the controlling expression for the rising time of unit hydrograph in large catchments, which are expressed in terms of the propagation time of maximum discharge or its delay time, respectively. When adopting the average value of 0.5 for parameters β, β' and β'', an approximate equation is obtained as follows:

$$t_{g\cdot l}=T_{g\cdot l}+0.5t_u \tag{7.31}$$

The rising time of unit hydrograph is given by this equation in large catchments for which the hydraulic meaning is clear from the above explanation.

When using the approximate equation of unit rainfall (6.14), the relation of eq.(3.3), $Q_{np}=\{\int_0^{t_d} r(t)dt/K_0\}^{1/p}$, and eq.(7.2), the peak of unit hydrograph is given by

$$q_{up\cdot l}=Q_{np}=C_c L_0 r_m=C_c L_0 R_{td}/t_d \tag{7.32}$$

where C_c satisfies

$$0<C_c=pt_d/(t_{pc\cdot l}-\varepsilon t_d)\leq 1$$

The value of C_c does not vary largely with rainfall and basin characteristics, and takes a value from 0.5 to 1 according to the observations in the United States[13]. When transforming R_{td} into $t_{pc\cdot l}$ by using eq.(7.2), the peak of unit hydrograph per unit area is expressed as follows:

$$q'_{up\cdot l}=C_c\{B_c/(t_{pc\cdot l}-\varepsilon t_d)\}^{1/\delta}/t_d \tag{7.33}$$

The transformation of $t_{pc\cdot l}$ into $T_{g\cdot l}$ by eq.(7.6) and use of 1/2 for β and β' gives

$$q'_{up\cdot l}=C_c\{B_c/(2T_{g\cdot l}-\varepsilon t_d)\}^{1/\delta}/t_d \tag{7.34}$$

In eqs.(7.33) and (7.34), the portion of $\{\ \}^{1/\delta}$ slightly decreases with an increase of B_c because $t_{pc\cdot l}$ and $T_{g\cdot l}$ increase with an increase of B_c. This is based on the empirical fact that this term is equivalent to the total rainfall R_{td} and the mean total rainfall in a catchment decreases with an increase of catchment area. As this term and C_c do not vary considerably[13], $q'_{up\cdot l}$ can be dominated by the rainfall duration t_d , decreasing with the increase of t_d.

Taylor and Schwarz[13] introduced the following relation from many observations:

$$q'_{up\cdot l}=Ce^{mt_d} \tag{7.35}$$

where C is a decreasing function of flow distance and m is a negative value which is a function of flow distance and gradient. They implied that $q_{up\cdot l}$ is mainly dominated by t_d and that it decreases with the increasing t_d. Their discussion is proved by the above hydraulic consideration. It should be noted that the function in eq.(7.34) has a shape similar to an exponential function in eq.(7.35) in the actual range of t_d, although $q'_{up\cdot l}$ is expressed as a hyperbolic function of t_d in eq.(7.34).

Snyder[10] proposed the following relation for a large catchment:

$$q'_{up\cdot l}=640\,C_p/\,T_{g\cdot l} \tag{7.36}$$

where C_p is a constant ranging from 0.56 to 0.69. The portion of $640C_p$ in the above equation corresponds to $C_p\{\ \}^{1/\delta}$ or C_pR_{td} in eqs.(7.33) and (7.34), respectively and then $T_{g\text{-}l}$ is a dominant factor. However, it goes without saying that t_d should be used instead of $T_{g\text{-}l}$, based on the discussion in this section. In a large catchment, $T_{g\text{-}l}$ varies in a linear proportion to t_d from eq.(7.8) and $T_{g\text{-}l}$ takes almost the same value as t_d in the United States[13]. Therefore, the use of t_d instead of $T_{g\text{-}l}$ is appropriate from a hydraulic viewpoint, even though Snyder's equation (7.36) bears practical significance.

When using $\beta''=1/2$ and $\epsilon=0.4$ in eqs.(7.29) and (7.2), the rising time of unit hydrograph is approximated by

$$t_{g\text{-}l}=\left(B_c/2R_{td}^{\delta}\right)+0.2t_d+0.5t_u \tag{7.37}$$

When $t_u=\eta t_d$ in a catchment, the difference between the rising times of unit hydrograph for rainfall conditions i and j is given by

$$t_{g\text{-}l_i}-t_{g\text{-}l_j}=(B_c/2)\left(1/R_{td_i}^{\delta}-1/R_{td_j}^{\delta}\right)+(0.2+0.5\eta)\left(t_{d_i}-t_{d_j}\right) \tag{7.38}$$

Since t_u is supposedly determined from $T_{g\text{-}l}$ in general and $T_{g\text{-}l}$ is assumed to be equal to t_d in the United States as mentioned above, η is in the range of $0<\eta<1$ and hence the value of 1/2~1/4 is reasonable for η. As shown in eq.(7.38), the variation of rising time of unit hydrograph caused by rainfall conditions is dominated by the total amount and duration of the rainfall, and the peak should be shifted backward according to the increasing rising time when the total amount of rainfall is small and its duration is long.

When neglecting the variation of C_c for the rainfall conditions i and j, the relationship between the peaks of unit hydrograph per unit area is given from eq.(7.33) as

$$\frac{q'_{up\text{-}l_i}}{q'_{up\text{-}l_j}}=\left(\frac{t_{pc\text{-}l_j}-\varepsilon t_{d_j}}{t_{pc\text{-}l_i}-\varepsilon t_{d_i}}\right)^{1/\delta}\frac{t_{d_i}}{t_{d_j}}=\frac{R_{t_{d_i}}}{R_{t_{d_j}}}\frac{t_{d_j}}{t_{d_i}} \tag{7.39}$$

When using the Talbot-type formula of $R_{td}=at_d/(b+t_d)$, eq.(7.39) reduces to

$$q'_{up\text{-}l_i}/q'_{up\text{-}l_j}=\left(b+t_{d_j}\right)/\left(b+t_{d_i}\right) \tag{7.40}$$

Note that the peak of unit hydrograph per unit area decreases with the increase of rainfall duration, but its variation is very small.

(4) Relationship between catchment scale and elements of unit hydrograph

As described in detail in the previous section, there is a significant difference in runoff mechanisms between small catchments as in Japanese rivers and large ones as in rivers of the United States. Therefore, different standpoints are needed to determine the rising time and the peak, which are important elements of unit hydrograph.

Based on the above discussions, the following conclusions are summarized:

i)As for the rainfall characteristics influencing the elements of unit hydrograph, the mean rainfall intensity around its peak is a dominant factor in small catchments, while the total amount and the duration, especially the latter, are controlling factors in large catchments. The present study has proved the difference of empirical facts between the United States and Japan for the application of unit hydrograph theory.

ii)The rising time of unit hydrograph can be determined from the delay time of maximum discharge in both of small and large catchments. However, it should be noted that there is a difference in the meaning of the delay time of maximum discharge between the two cases. There is a significant difference in the peaks of unit hydrograph between small and large catchments. Therefore, a considerably lower value than the maximum discharge by an actual rainfall should be adopted in the former case, while almost the same value in the latter. This is the main reason why the total amount of unit rainfall is chosen to be much smaller than that of actual rainfall in Japan, while the larger value such as 1 inch in the United States.

iii)The variation of the elements of unit hydrograph is mainly caused by the variation of rainfall conditions. The elements of unit hydrograph vary according to the rainfall conditions more considerably in small catchments than in large ones. In particular, the peak of unit hydrograph varies significantly in small catchments. The conventional unit hydrograph theory has been widely used in the United States without any revision. On the other hand, many difficulties have been involved in the application of unit hydrograph theory in Japan. This is because the difference of runoff mechanisms between small and large catchments has not been well recognized in Japan.

iv)The equations representing the elements of unit hydrograph derived in this chapter are of practical use. In particular, the equations which give the change of the elements of unit hydrograph caused by the variation of rainfall conditions appear to be advantageous when applying the unit hydrograph method in catchments where available discharge data are scarce. It is needless to say that the equations developed in the present paper give an important direction to the future studies on synthetic unit hydrograph.

8. ERRORS DUE TO THE APPLICATION OF THE UNIT HYDROGRAPH METHOD

It is easily anticipated that some errors might accrued from the application of the unit hydrograph method based on the hypothesis of linearity to the runoff process with nonlinear characteristics in nature. This chapter deals with the application errors in the runoff analysis through the unit hydrograph resulting from the use of unit rainfall. Such results will have an insight into the limitation for the application of the unit hydrograph method as well as some guidelines to the synthetic unit hydrograph approach.

(1) Error of maximum discharge resulting from use of unit hydrograph method

The elements of unit rainfall and unit hydrograph which generate the peak discharge have been discussed in the previous two chapters. In general, it is extremely difficult to derive the functional relationship from arbitrary rainfall patterns even if it gained the satisfactory results, because the rainfall conditions are implicitly included in these relationships. The errors of maximum discharge and lag time differ from a small catchment to a large one. These errors can be computed from the following two steps: in the first place, the unit hydrograph is derived from the use of corresponding unit rainfall (r_u, t_u) with average intensity $r_{m\cdot u}$ and duration time $t_{d\cdot u}^{*}$ and the second step is to compute the hydrograph by superposing the above unit hydrogaph on the general rainfall pattern with average intensity r_m and duration t_d.

<u>(a) The case of a small catchment $(t_u<t_c,\ t_d>t_c)$:</u> As described in 6.(1), $(\alpha_{tp})_m$ of unit rainfall and a general rainfall pattern are given by eqs. (6.1) and (6.2), respectively. The peak discharge calculated from the unit hydrograph method can coincide with the observed one when the unit rainfall (r_u, t_u) satisfied with $(\alpha_{tp})_{m\cdot u}= (\alpha_{tp})_{m\cdot n}$ is applied. The values of r_u and t_u as derived above can be valid only in the case of average rainfall intensity $r_{m\cdot u}$ within a fixed propagation time even in a particular catchment, but are not applicable to all hyetographs.

The relative error of propagation time, γ_t can be defined by the ratio of propagation times as follows:

$$\gamma_t = \frac{(\alpha_{tp})_{m\cdot u}}{(\alpha_{tp})_{m\cdot n}} = \frac{B_c^p(r_u t_u)^{\delta}}{p^p r_m^{\epsilon}(B_c + \epsilon r_u^{\delta} t_u^{\delta+1})} \tag{8.1}$$

Equation (8.1) implies that the error becomes zero for γ_t=1 and will increase with γ_t deviating from 1.

---

* The notations, r_{mp}, r_m and t_d which are described for the elements of unit rainfall and unit hydrograph in the previous two chapters are synthesized to the notations, $r_{m\cdot u}$ and $t_{d\cdot u}$ in this chapter.

The set of r_u and t_u in the unit rainfall is valid only in the special case of average rainfall intensity $r_{m\cdot u}$ and its relationship is given by eq.(6.5) as follows:

$$r_u=\left(p^{P}B_c r_{m\cdot u}^{\epsilon}\right)^{1/\delta}\Big/\left\{t_u\left(B_c^{P}-\epsilon p^{P} t_u r_{m\cdot u}^{\epsilon}\right)^{1/\delta}\right\}$$

Substituting the above relation into eq.(8.1) yields

$$\gamma_t=\left(r_{m\cdot u}/r_m\right)^{\epsilon}=\kappa^{\epsilon} \tag{8.2}$$

where

$$\kappa=r_{m\cdot u}/r_m$$

ϵ is equal to 0.4 from Manning's resistance law. It follows from ep.(8.2) that the relative error of propagation time γ_t, nearly equals the ratio of average rainfall intensity $r_{m\cdot u}$ of the unit hydrograph to the average intensity of rainfall r_m to the power 0.4.

The relationship between the coefficients of discharge α_q and propagation α_t is given by eq.(4.5). Therefore, the relative error γ_q of $(\alpha_{pq})_m$ is also computed by the same procedure as in γ_t as follows:

$$\gamma_q=\left(\alpha_{qp}\right)_{m\cdot u}\Big/\left(\alpha_{qp}\right)_{m\cdot n}=\left(\gamma_{m\cdot u}/\gamma_m\right)^{\epsilon}=\kappa^{\epsilon} \tag{8.3}$$

The error $\sigma_{t(q)}$ in terms of the propagation time (or discharge) is expressed as

$$\begin{aligned}\kappa<1\ ;\quad & \sigma_{t(q)}=1-\gamma_{t(q)}=1-\kappa^{\epsilon}\\ \kappa>1\ ;\quad & \sigma_{t(q)}=1-1/\gamma_{t(q)}=1-1/\kappa^{\epsilon}\end{aligned} \tag{8.4}$$

For example, the error $\sigma_{t(q)}$ becomes 10 % for the case of $\gamma_{t(q)}$=0.9. **Fig. 8.1** shows the relation of

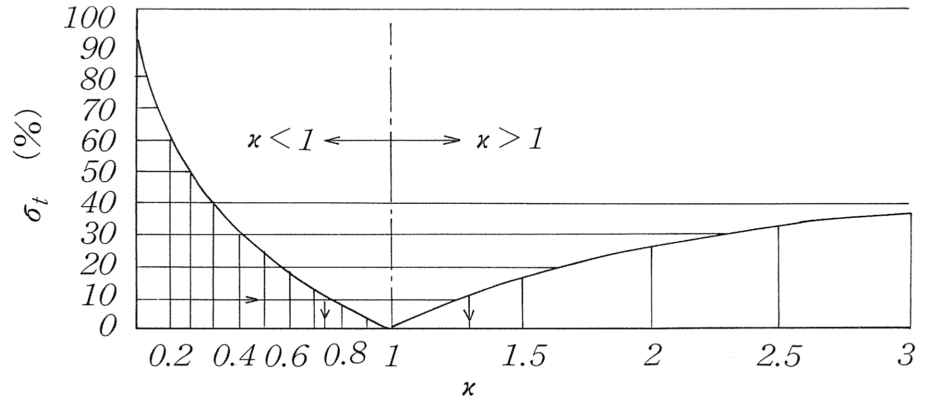

Fig. 8.1 Relation between σ_t and κ

eq.(8.4) in the case of ϵ=0.4. The range of r_m corresponding to $\sigma_{t(q)}$ less than 10% can be obtained through the following procedure. First, the horizontal line should be drawn from point $\sigma_{t(q)}$ and second, the vertical lines down from the points of intersection with the curves depending on κ<1 and κ>1, give the lower value of 0.75 and upper value of 1.3. Finally, the range of r_m for $\sigma_{t(q)}$ less than 10% can easily be defined.

Although the errors of coefficients α_t and α_q for propagation time and discharge are discussed above, the error of propagation time is not an adequate index to the occurrence time of maximum discharge. Even though the occurrence condition of maximum discharge is given by $r(\tau_p)=r(t_p)$, τ_p is not held fixed, when considering the error of propagation time $(t_p-\tau_p)$ of maximum discharge in the case that the average intensity of actual rainfall, r_m is not equal to $r_{m\cdot u}$.

Assuming that the gradients of hyetograph close to $r(\tau_p)$ and $r(t_p)$ are almost the same, the difference of occurrence time of maximum discharge becomes nearly a half the difference of propagation time, because the relation of $|t_{p\cdot u}-t_{p\cdot n}|\fallingdotseq|\tau_{p\cdot u}-\tau_{p\cdot n}|$ is realized from the condition of $r(\tau_p)=r(t_p)$ (see **Fig. 8.2**). When ϕ is defined as the difference between the occurrence time of maximum discharge with use of the unit hydrograph method, $t_{p\cdot u}$, and the acutual one, $t_{p\cdot n}$, is expressed as

$$\phi = t_{p\cdot u} - t_{p\cdot n} = \pm\frac{1}{2}\left|(t_{p\cdot u}-\tau_{p\cdot u})-(t_{p\cdot n}-\tau_{p\cdot n})\right| \tag{8.5}$$

where the sign denotes the plus in the case of $\kappa<1$ and the minus in the case of $\kappa>1$.
The following relation is obtained, using the ratio of propagation times, γ_t:

$$(t_{p\cdot u}-\tau_{p\cdot u})/(t_{p\cdot n}-\tau_{p\cdot n})=1/\gamma_t$$

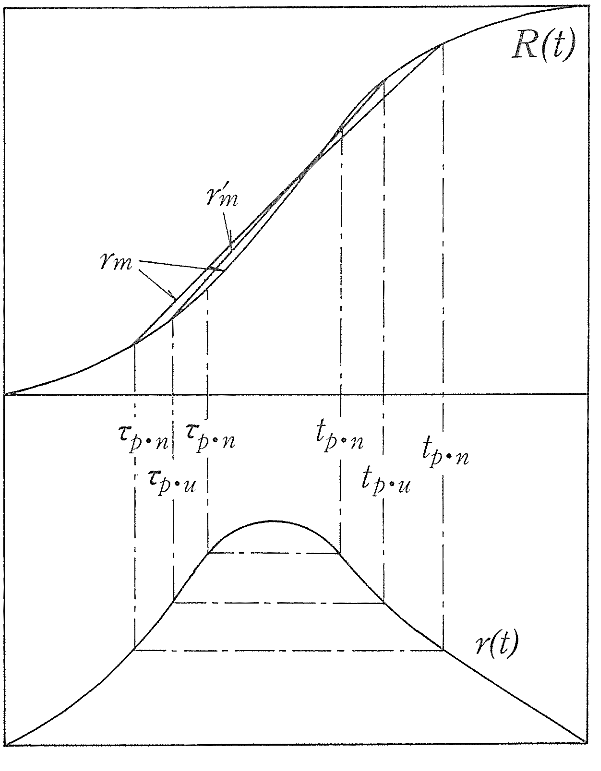

Fig. 8.2 Relation between rainfall mass curve and starting and reaching times of rainfall disturbance

Substitution of the above relation into eq.(8.5) gives

$$\phi=\frac{1}{2}(t_{p\cdot n}-\tau_{p\cdot n})(1/\gamma_t-1)=\frac{1}{2}(t_{p\cdot u}-\tau_{p\cdot u})(1-\gamma_t) \tag{8.6}$$

ϕ is divided by $t_{p\cdot u}-\tau_{p\cdot u}=K_0L_0^p/r^{\epsilon}_{m\cdot u}$ to yield a nondimensional number. As a result, the error of occurrence time of maximum discharge is given by

$$\psi=\left(r^{\epsilon}_{m\cdot u}/K_0L_0^{p}\right)\phi=\left(1-\gamma_t\right)/2 \tag{8.7}$$

Note that $\psi=0$ if $\gamma_t=1$ and ϕ increases as K_0 and L_0 increase with the constant value of ψ. The index of $t_{p\cdot u}/t_{p\cdot n}$ is not included in the description of the error of occurrence time of maximum discharge, because the value of $t_{p\cdot u}-t_{p\cdot n}$ varies with the occurrence time of peak rainfall, even if the difference of occurrence time of maximum discharge is identical.

Let us next consider the error of maximum discharge. The maximum discharge $Q_{n\cdot p}$ generated by rainfall with average intensity r_m is rewritten by eq.(5.8) as

$$Q_{n\cdot p}=\left\{(\alpha_{qp})_{m\cdot n}/K_0^{1/p}\right\}\int_{\tau_{p\cdot n}}^{t_{p\cdot n}}r(t)dt$$

On the other hand, the maximum discharge $Q_{u\cdot p}$ with the unit hydrograph method in which the unit rainfall $(r_u,\ t_u)$ has average intensity $r_{m\cdot u}$, is represented by eq.(5.9) as follows:

$$Q_{n\cdot p}=\frac{(\alpha_{qp})_{m\cdot u}}{K_0^{1/p}}\left\{m_1\int_{\tau_{p\cdot u}}^{t_{ui=1}}r_u dt+\sum_{i=2}^{i}m_i\int_0^{t_u}r_u dt+m_{i+1}\int_{t_p-t_{u_{i=i}}}^{t_{p\cdot u}}r_u dt\right\}=\frac{(\alpha_{qp})_{m\cdot u}}{K_0^{1/p}}\int_{\tau_{p\cdot u}}^{t_{p\cdot u}}r(t)dt$$

Then, the relative error γ_Q between the maximum discharge with the unit hydrograph method and the actual one is obtained by

$$\gamma_Q=\frac{Q_{u\cdot p}}{Q_{n\cdot p}}=\frac{(\alpha_{qp})_{m\cdot u}\int_{\tau_{p\cdot u}}^{t_{p\cdot u}}r(t)dt}{(\alpha_{qp})_{m\cdot n}\int_{\tau_{p\cdot n}}^{t_{p\cdot n}}r(t)dt}=\kappa^{\epsilon}\frac{t_{p\cdot u}-\tau_{p\cdot u}}{t_{p\cdot n}-\tau_{p\cdot n}}\frac{r_m'}{r_m}=\frac{r_m'}{r_m} \tag{8.8}$$

where

$$r_m'=\int_{\tau_{p\cdot u}}^{t_{p\cdot u}}r(t)dt\Big/\int_{\tau_{p\cdot u}}^{t_{p\cdot u}}dt,\quad r_m=\int_{\tau_{p\cdot n}}^{t_{p\cdot n}}r(t)dt\Big/\int_{\tau_{p\cdot n}}^{t_{p\cdot n}}dt$$

As mentioned earlier, the propagation time of maximum discharge, $(t_{p\cdot u}-\tau_{p\cdot u})$ is specified by $r_{m\cdot n}\cdot(t_{p\cdot u}-\tau_{p\cdot u})=(t_{p\cdot n}-\tau_{p\cdot n})$ for $r_m=r_{m\cdot u}$ and thus $\gamma_Q=1$. In other words, it is obvious that the error of maximum discharge with the unit hydrograph method becomes zero.

As the relation of $r(\tau_p)=r(t_p)$ can hold between τ_p and t_p, both of $r(\tau_p)$ and $r(t_p)$ decrease with an increase of (r_p-t_p) and increase with a decrease of (r_p-t_p) under the condition that $r(t)$ changes with time. Therefore, the following relationship can be realized:

$$r_m > r_{m\cdot u} \rightarrow t_{p\cdot n} - \tau_{p\cdot n} < t_{p\cdot u} - \tau_{p\cdot u} \rightarrow r_m > r_m^{\prime} \rightarrow \gamma_Q < 1$$
$$r_m < r_{m\cdot u} \rightarrow t_{p\cdot n} - \tau_{p\cdot n} > t_{p\cdot u} - \tau_{p\cdot u} \rightarrow r_m < r_m^{\prime} \rightarrow \gamma_Q > 1 \qquad (8.9)$$

The above relation indicates that the maximum discharge by the unit hydrograph method becomes small for $\kappa<1$ and large for $\kappa>1$, when compared with the actual one.

Besides eq.(8.8), additional explanations are herein introduced to facilitate the understanding of γ_Q. The assumption of $|t_{p\cdot u}-t_{p\cdot n}|=|\tau_{p\cdot u}-\tau_{p\cdot n}|$ is used to derive the error of occurrence time of maximum discharge. With reference to Fig. 8.2, $r(t_{p\cdot n})$ and $r(\tau_{p\cdot n})$ are defined as the rainfall intensities for the periods from $t_{p\cdot u}$ to $t_{p\cdot n}$ and from $\tau_{p\cdot u}$ to $\tau_{p\cdot n}$, respectively. It is expected that $r(t_{p\cdot n})=r(\tau_{p\cdot n})$ from the occurrence condition of maximum discharge. When using the above assumption and approximation, the following equation is obtained:

$$r_m^{\prime} = \frac{r_m(t_{p\cdot n}-\tau_{p\cdot n})}{t_{p\cdot u}-\tau_{p\cdot u}} \pm \frac{2r(t_{p\cdot n})|t_{p\cdot u}-t_{p\cdot n}|}{t_{p\cdot u}-\tau_{p\cdot u}} \qquad (8.10)$$

where the sign denotes the plus when $\kappa<1$ and the minus when $\kappa>1$.

As a result, eq.(8.8) reduces to

$$\gamma_Q = \frac{r_m^{\prime}}{r_m} = \frac{t_{p\cdot n}-\tau_{p\cdot n}}{t_{p\cdot u}-\tau_{p\cdot u}} \pm \frac{2r(t_{p\cdot n})}{r_m}\frac{t_{p\cdot u}-t_{p\cdot n}}{t_{p\cdot u}-\tau_{p\cdot u}} = \gamma_t + 2\nu\psi = \gamma_t + \nu(1-\gamma_t) \qquad (8.11)$$

where

$$\nu = r(t_{p\cdot n})/r_m \leq 1$$

The relation of $|1-\gamma_Q| \leq |1-\gamma_t|$ is easily developed through the above equation. It is understood that the relative error of maximum discharge deviated from 1 is smaller than that of propagation time. However, it is too hasty to conclude that the drawback of unit hydrograph method is caused by the larger error of propagation time than that of maximum discharge, even though the characteristics and magnitudes of errors are of practical concern. For the case of γ_t=0.9 and ν=0.5, γ_Q becomes 0.95 from eq.(8.11) and the difference between the occurrence times of maximum discharge is computed from eq.(8.6) as

$$\phi = \frac{1}{2}(t_{p\cdot n}-\tau_{p\cdot n})(1/0.9-1) = \frac{1}{18}(t_{p\cdot n}-\tau_{p\cdot n})$$

On the other hand, the difference between the maximum discharges is given by

$$Q_{u\cdot p}-Q_{n\cdot p} = Q_{n\cdot p}(\gamma_Q-1) = -0.05\,Q_{n\cdot p}$$

When setting $t_{p\cdot n}-\tau_{p\cdot n}$ as 9 hr and $Q_{n\cdot p}$ as 2,000 m^3/s for actual flood runoff situations, the following relations are obtained:

$$\phi = 0.5\,hr\ , \quad Q_{u\cdot p}-Q_{n\cdot p} = -100\ m^3/s$$

This particular example shows that the error of maximum discharge is much larger than that of occurrence time of maximum discharge. Of practical interests is how to estimate the maximum discharge as accurately as possible, even though the propagation time is an important issue in the light of error theory.

When the rainfall intensity is constant (i.e., $r_m=r(t_{p\cdot n})$), $\nu=1$. Although the occurrence time of maximum discharge changes with eq. (8.6), the error of maximum discharge itself becomes zero. In general, as ν is quite smaller than 1, the second-term of the right hand side in eq.(8.11) can be neglected for ensuring safety as follows:

$$\gamma_Q \fallingdotseq \gamma_t \qquad \textbf{(8.12)}$$

The parameter ν expresses the time variation of rainfall. When the variation is small, both of ν and γ_Q approach 1. In other words, the accuracy of unit hydrograph method increases as the pattern of hyetograph becomes flatter.

The error of maximum discharge is summarized as follows:

$$\left.\begin{array}{ll} \gamma_Q<1\ ; & \sigma_Q=1-\gamma_Q=(1-\nu)(1-\gamma_t) \\ \gamma_Q>1\ ; & \sigma_Q=1-1/\gamma_Q=1-1/\{\gamma_t+\nu(1-\gamma_t)\} \end{array}\right\} \qquad \textbf{(8.13)}$$

This equation has the same meaning as for $\sigma_{t(q)}$. The relationship between γ_t and κ is given by eq.(8.2), which is graphically expressed in **Fig. 8.3**. Region I is derived from the use of eq.(8.2), while II from eq.(8.11) with parameter ν, and III is drawn from eq.(8.13). The error of maximum discharge is estimated from this figure in a following way: γ_t is first obtained through κ as illustrated with an arrow line, γ_Q with parameter ν in the second step and σ_Q is finally obtained. On the contrary, the upper and lower limits of r_m for the application of unit hydrograph method can be estimated by reversely tracking against the above arrow line, when the value of γ_Q is intended to be constrained within a given percentage point.

The above statement can analytically be solved. First of all, as $\gamma_t=1-\sigma_Q/(1-\nu)$ from the upper portion of eq.(8.13), the upper limit r_m' of r_m is calculated by

$$r_m'=r_{m\cdot n}/\{1-\sigma_Q/(1-\nu)\}^{1/\epsilon} \qquad \textbf{(8.14)}$$

Moreover, as $\gamma_t=\{1-\nu(1-\sigma_Q)\}/\{(1-\nu)(1-\sigma_Q)\}$ from the lower portion of eq.(8.13), the lower limit r_m'' is formulated as follows:

$$r_m''=r_{m\cdot u}[(1-\nu)(1-\sigma_Q)/\{1-\nu(1-\sigma_Q)\}]^{1/\epsilon} \qquad \textbf{(8.15)}$$

$$r_m'=1.75\,r_{m\cdot n}\ , \qquad r_m''=0.35\,r_{m\cdot n}$$

For example, when setting σ_Q as 10% , ν as 0.5 and ϵ as 0.4 , the following values are obtained: It is, therefore, found from this particular condition that the unit hydrograph method is applicable only to the case where r_m is in a range between 0.35 $r_{m \cdot u}$ and 1.75 $r_{m \cdot u}$ for rainfall intensity $r_{m \cdot u}$ selected as the basis of unit hydrograph. It is rather difficult to comprehend the physical meaning of $\nu = r(t_{p \cdot n})/r_m$. Hence, an attempt is made to approximate the hyetograph by a triangular pattern as

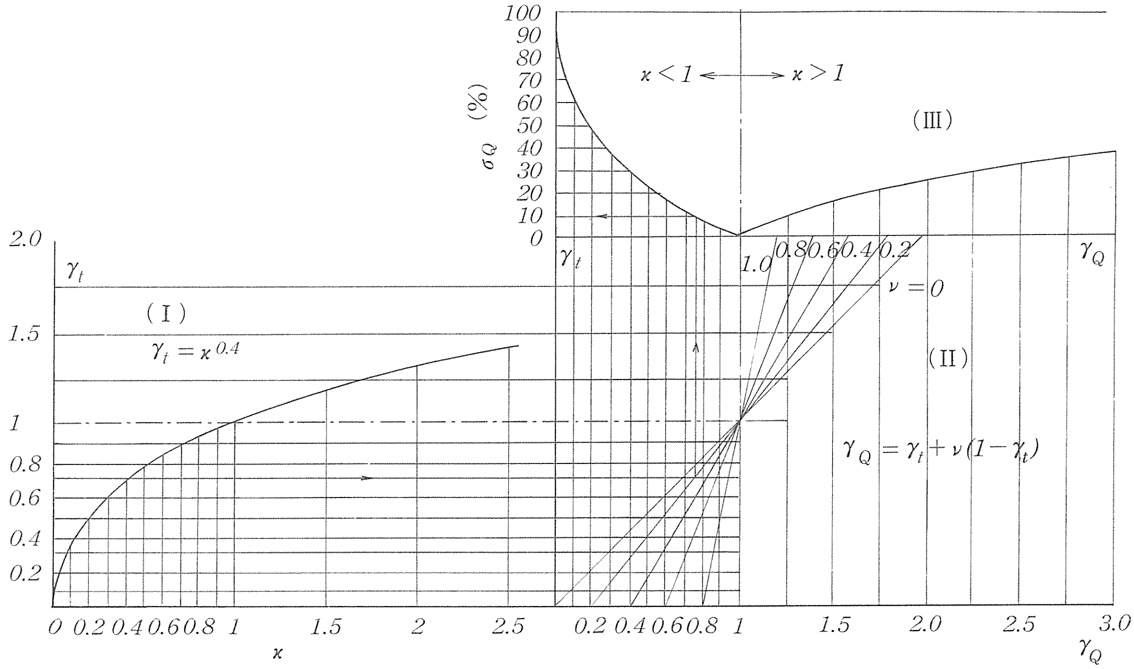

Fig. 8.3 Relation between σ_Q and κ, ν

shown in **Fig. 8.4**. Such an approximation facilitates the understanding of physical meaning of ν. t_{pc} denotes the propagation time of maximum discharge, while r_p the maximum rainfall intensity. As shown in **Fig.8.4**, the following equations are easily obtained:

$$r_m = \{r_p + r(t_{p \cdot n})\}/2 \tag{8.16}$$

$$r(t_{p \cdot n}) = r_p \tau_{p \cdot n}/t_{r1} = r_p(t_{r1} - t_{c1})/t_{r1} \tag{8.17}$$

From the geometrical property of a triangular hyetograph, the following relation can hold:

$$t_{r1} + t_{r2} = t_d, \quad t_{c1} = t_{r1} t_{pc}/t_d$$

Substituting the above expression into eq.(8.17) gives

$$r(t_{p \cdot n}) = r_p(1 - t_{pc}/t_d)$$

Therefore, ν is expressed by t_{pc} and t_d via eq. (8.7) as

$$\nu = r(t_{p \cdot n})/r_m = 2(t_d - t_{pc})/(2t_d - t_{pc}) \tag{8.18}$$

t_d is directly calculated from rainfall data and t_{pc} can be obtained through the approximate method in terms of occurrence condition of maximum discharge as described in 2.(2)c. The values of ν and r_m for each rainfall event are estimated from the calculated values of t_d and t_{pc} and then the error σ_Q in

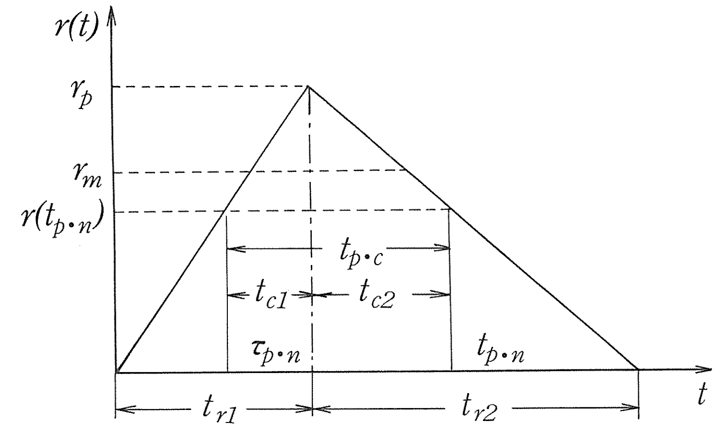

Fig. 8.4 Triangular model distribution of rainfall illustrating the meaning of ν

the application of unit hydrograph method is obtained by eq.(8.11). On the contrary, the applicable range of rainfall intensity r_m for a given value of σ_Q is also calculated by eqs.(8.14) and (8.15).

In short, it is necessary to consider not only rainfall intensity but also its time distribution, even when the application limit of unit hydrograph method in a single catchment is investigated. Therefore, the issues with respect to the application limit of unit hydrograph method in a small catchment such as in Japanese rivers include the three factors of basin characteristics B_c, average intensity of rainfall r_m and parameter ν denoting its time variation. It is easily understood from the above discussion that the application of unit hydrograph method in a small catchment involves difficulty and complexity in practices.

(b) The case of a large catchment $(t_u<t_c,\ t_d<t_c)$ As described in 6.(2), $(\alpha_{tp})_m$ for the unit rainfall and the actual rainfall are formulated by eqs.(6.1) and (6.12), respectively, and the relative error of propagation time γ_t, is represented by

$$\gamma_t=\frac{(\alpha_{tp})_{m\cdot u}}{(\alpha_{tp})_{m\cdot n}}=\left(\frac{B_c+\varepsilon r_m^{\sigma}t_d^{\sigma+1}}{B_c+\varepsilon r_u^{\sigma}t_u^{\sigma+1}}\right)\left(\frac{r_u t_u}{r_m t_d}\right)^{\sigma} \tag{8.19}$$

The values of r_u and t_u are valid only for the specified rainfall with average intensity $r_{m\cdot n}$ and duration time $t_{d\cdot u}$ and their relation is given by $r_u t_u=r_{m\cdot u}t_{d\cdot u}$. By substituting this relation into eq.(8.19) and considering that $B_c \gg \varepsilon r_m^{\delta} t_d^{\delta+1}$ and $B_c \gg \varepsilon r_u^{\delta} t_u^{\delta+1}$, the following relationship is finally obtained:

$$\gamma_t=(r_{m\cdot u}t_{d\cdot u}/r_m t_d)^{\delta}=(R_u/R_{td})^{\delta}=\mu^{\delta} \tag{8.20}$$

where R_u is the total amount of unit rainfall, R_{td} is the total amount of actual rainfall, and its ratio is denoted by μ. Thus, the relative error γ_q of discharge coeficient $(\alpha_{pq})_m$ is also represented as follows:

$$\gamma_q=(\alpha_{p\cdot q})_{m\cdot u}/(\alpha_{p\cdot q})_{m\cdot n}=\mu^{\delta} \tag{8.21}$$

The relative errors γ_t and γ_q relating to the coefficients of propagation time* and discharge in terms of maximum discharge are proportional to $\mu=R_u/R_{td}$ to the power of about 2/3. The following relationship is derived for δ=2/3:

$$\left.\begin{aligned} \mu<1\ ;\quad \sigma_{t(q)}&=1-\gamma_{t(q)}=1-\mu^{2/3} \\ \mu>1\ ;\quad \sigma_{t(q)}&=1-1/\gamma_{t(q)}=1-1/\mu^{2/3} \end{aligned}\right\} \tag{8.22}$$

Equation (8.22) gives the errors on propagation time (occurrence time) and coefficients of maximum discharge. **Fig. 8.5** illustrates the relation of eq.(8.22). As in **Fig.8.3**, for example, the upper limit for $\sigma_{t(q)}$=10% becomes 1.2, while the lower 0.85. This result specifies the applicable range of R_{td}.

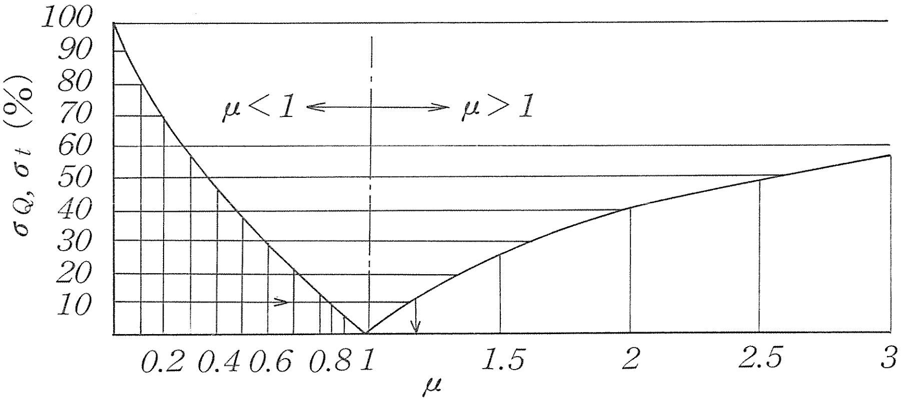

Fig. 8.5 Relation between σ_t, σ_Q and μ

Since $Q_p=\{(\alpha_{qp})_m/K_0^{1/p}\}\int_0^{t_d} r(t)dt$ with respect to the maximum discharge, the relative error γ_Q is deduced through the same procedure as in a preceding case as follows:

$$\gamma_Q=\frac{Q_{u\cdot p}}{Q_{n\cdot p}}=\frac{(\alpha_{qp})_{m\cdot u}}{(\alpha_{qp})_{m\cdot n}}\int_0^{t_d} r(t)dt\Big/\int_0^{t_d} r(t)dt=\mu^{\delta} \tag{8.23}$$

When setting μ as 2/3, the error σ_Q is given by

$$\left.\begin{aligned} \mu<1\ :\quad \sigma_Q&=1-\gamma_Q=1-\mu^{2/3} \\ \mu>1\ :\quad \sigma_Q&=1-1/\gamma_Q=1-1/\mu^{2/3} \end{aligned}\right\} \tag{8.24}$$

As $\sigma_Q=Q_{t(q)}$, this relationship is represented in **Fig. 8.5**. The upper and lower limits of μ corresponding to error σ_Q provide the applicable range of R_{td}.

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* This is different from the case of a small catchment and τ_p is always zero. Hence, γ_t in terms of propagation time is assumed to be equal to the occurrence time of maximum discharge.

The above discussions indicate that the total amount of rainfall is a deciding factor for controlling the applicable condition of unit hydrograph method in a large catchment, which significantly differ from that in a small catchment. In other words, The application of unit hydrograph method to a large catchment turns out to be much effective, compared with that in a small catchment.

(2) Error of runoff hydrograph resulting from use of unit hydrograph method

In the previous section, the error of unit hydrograph method near the peak of hydrograph has been discussed in detail. Such results cannot simply be applied to the whole runoff process, because α_t changes with the time interval in the runoff machanism. To remedy this situation, a different approach is pursued to quantify the error of hydrograph (the whole runoff process) by superposing the unit hydrograph which is derived from unit rainfall (r_u, t_u) to make the maximum discharge coincide by the use of unit hydrograph method.

(a) The case of a small catchment $(t_u<t_c,\ t_d>t_c)$: The runoff mechanism for this case is schematically illustrated in **Fig. 8.6**. The discussion in the previous section (a) is valid for the region of $t_c \le t \le t_d$ and it is needless to say that the peak occurs in this region. The runoff mechanism is classified according to the difference of α_t as follows:

$$(A)\ 0\le t<t_u\ ,\quad (B)\ t_u\le t<t_c\ ,\quad (C)\ t_c\le t<t_d\ ,\quad (D)\ t_d\le t$$

The error in each region is summarized below.

(A) $0\le t<t_u$: When assuming that B_c' is the modified parameter of L_0 in B_c, which is replaced by the distance of x_0' measured from the lower end to the upper end, the following equations are obtained from eq.(5.6):

$$(\alpha_t)_{m\cdot u}=p^{\,p}(B_c')^{\epsilon}r_u^{\epsilon}\ ,\quad (\alpha_t)_{m\cdot n}=p^{\,p}(B_c')^{\epsilon}r_m^{\epsilon}$$
$$\gamma_t=(\alpha_t)_{m\cdot u}/(\alpha_t)_{m\cdot n}=(r_u/r_m)^{\epsilon}$$

Substitution of eq.(6.6) for r_u into the above equation yields

$$\gamma_t=(p^{p2}B_c^{\epsilon p}/t_u^{\epsilon})(r_{m\cdot u}^{p}/r_m)^{\epsilon} \tag{8.25}$$

When defining Q_u and Q_n as the discharge by the unit hydrograph method and the actual discharge, respectively, they are expressed as

$$Q_u=\frac{(\alpha_q)_{m\cdot u}}{K_0^{1/p}}\frac{\int_0^{t_u}r(t)dt}{\int_0^{t_u}r_u dt}\int_0^{t}r_u dt=\frac{(\alpha_q)_{m\cdot u}}{K_0^{1/p}}\int_0^{t}r_m' dt$$

$$Q_n\fallingdotseq\frac{(\alpha_q)_{m\cdot n}}{K_0^{1/p}}\int_0^{t}r_m dt$$

Therefore, the following relation is deduced:

$$\gamma_Q = Q_u/Q_n \fallingdotseq \gamma_t r'_m/r_m \tag{8.26}$$

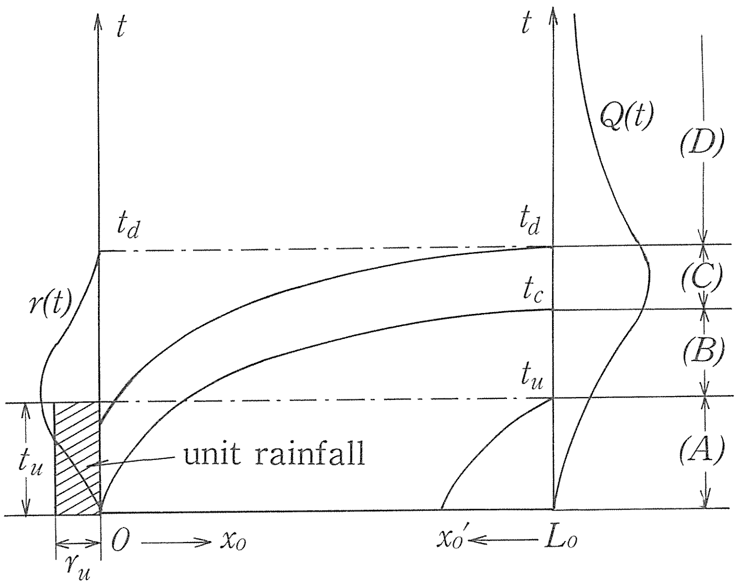

Fig. 8.6 Runoff mechanism in the case of $t_u < t_c$ and $t_d > t_c$

where

$$r'_m = \int_0^{t_u} r(t)dt \Big/ \int_0^{t_u} dt\ , \quad r_m = \int_0^{t} r(t)dt \Big/ \int_0^{t} dt$$

As r_m' is nearly equal to r_m when t_u is not large, the following equation is obtained with p=0.6:

$$\gamma_Q \fallingdotseq \gamma_t = \left(0.6^{0.36} B_c^{0.24} / t_u^{0.4}\right)\left(r_{m\cdot n}^{0.6}/r_m\right)^{0.4} \tag{8.27}$$

Furthermore, as the value in the first parenthesis of the right-hand side is nearly equal to 1, γ_Q may be approximated by

$$\gamma_Q = \left(r_{m\cdot n}^{0.6}/r_m\right)^{0.4} \tag{8.28}$$

<u>(B) $t_u \leq t < t_c$:</u> The use of eqs.(5.7) and (5.6) yields

$$(\alpha_t)_{m\cdot u} = B'_c (r_u t_u)^{\delta} \Big/ \left\{B'_c + \epsilon r_u^{\delta} t_u^{\delta+1}\right\}\ , \quad (\alpha_t)_{m\cdot n} = p^{\,p} B_c'^{\epsilon} r_m^{\epsilon}$$

When defining r_u as the ratio of the above equations, which is substituted into eq.(6.6), the following relation is obtained:

$$\gamma_t = (\alpha_t)_{m\cdot u} / (\alpha_t)_{m\cdot n} \fallingdotseq (B_c / B'_c)^{\epsilon} (r_{m\cdot u}/r_m)^{\epsilon} \tag{8.29}$$

Moreover, the following equations are derived:

$$Q_u \fallingdotseq \left\{(\alpha_q)_{m\cdot u}/K_0^{1/p}\right\}\sum_{i=0}^{i}\int_0^{t_u} r_u dt + m_{i+1}\int_0^{t-it_u} r_u dt$$

$$\fallingdotseq \left\{(\alpha_q)_{m\cdot u}/K_0^{1/p}\right\}\int_0^{t} r_m dt$$

$$Q_n \fallingdotseq \left\{(\alpha_q)_{m\cdot n}/K_0^{1/p}\right\}\int_0^{t} r_m dt$$

Where i satisfies the relation of $it_u<t<(i+1)t_u$ and

$$m_i = \left[\int_0^{t_u} r(t)dt\right]_i \Big/ \int_0^{t_u} r_u dt \ , \quad r_m = \int_0^{t} r(t)dt \Big/ \int_0^{t} dt$$

When setting ϵ as 0.4, γ_Q is reduced to

$$\gamma_Q = Q_u/Q_n = \gamma_t = (B_c/B_c')^{0.4}(R_{m\cdot n}/r_m)^{0.4} \tag{8.30}$$

Since $B_c>B_c'$ and generally $r_{m\cdot u}>r_m$, the discharge resulting from the use of unit hydrograph method becomes larger than the actual one. The smaller B_c' is, or the closer t approaches t_u, the larger the relative error becomes. Moreover, the closer B_c' approaches B_c, or the closer t approaches t_c, the smaller the relative error becomes.

(C) $t_c \le t < t_d$: As $(\alpha_t)_m$ in this region is represented by the same form as in (a) of the previous section, the following relationship can hold:

$$\gamma_t = \left(r_{m\cdot u}/r_m\right)^{\epsilon} \tag{8.31}$$

where

$$r_m = \int_{\tau}^{t} r(t) \Big/ \int_{\tau}^{t} dt$$

γ_Q denoting the relative error at the same time t is also given by

$$\gamma_Q = \frac{(\alpha_q)_{m\cdot u}}{(\alpha_q)_{m\cdot n}}\int_{\tau_n}^{t} r(t)dt \Big/ \int_{\tau_n}^{t} r(t)dt = \gamma_t \int_{\tau_u}^{t} r(t)dt \Big/ \int_{\tau_n}^{t} r(t)dt \tag{8.32}$$

where τ_u is the starting time of propagation of unit rainfall and expressed as $t-\tau_u=t_{p\cdot u}-\tau_{p\cdot u}$. As τ_u is the starting time of propagation of the actual rainfall, the following relation is obtained:

$$\begin{cases} \gamma_t>1 \ : \ \int_{\tau_u}^{t} r(t)dt \Big/ \int_{\tau_n}^{t} r(t)dt<1 \\ \gamma_t<1 \ : \ \int_{\tau_u}^{t} r(t)dt \Big/ \int_{\tau_n}^{t} r(t)dt>1 \end{cases}$$

Therefore, since $|1-\gamma_Q|\le|1-\gamma_t|$, the relative error deviated from 1 on the discharge is generally smaller than that on the propagation time. As $\tau_u \fallingdotseq \tau_n$, the following result guarantees the safety side, when setting ϵ as 0.4:

$$\gamma_Q \fallingdotseq \gamma_t = \left(r_{m\cdot u}/r_m\right)^{0.4} \tag{8.33}$$

(D) $t_d \le t$: The following relation is formulated through eq. (5.7):

$$(\alpha_t)_{m\cdot u}=B_c(r_u t_u)^{\delta}\Big/\left\{B_c+\epsilon r_u^{\delta} t_u^{\delta+1}\right\}$$
$$(\alpha_t)_{m\cdot n}=B_c\{r_m(t_d-\tau)\}^{\delta}\Big/\left\{B_c+\epsilon r_m^{\delta}(t_d-\tau)^{\delta+1}\right\}$$

where $r_m=\int_{\tau}^{t_d}r(t)dt/\int_{\tau}^{t_d}dt$, τ is the starting time of propagation and the condition of $t_d-\tau>t_u$ is satisfied.

The propagation starting from τ in the interval of $t_d-\tau\le t_u$ can be neglected because of a small value of t_u. Substituting r_u from eq.(6.6) into the ratio of the above two equations yields

$$\gamma_t=\frac{p^p}{B_c^{1-\epsilon}}\frac{B_c+\epsilon r_m^{\delta}(t_d-\tau)^{\delta+1}}{(t_d-\tau)^{\delta}}\frac{r_{m\cdot u}^{\epsilon}}{r_m^{\delta}}\fallingdotseq\frac{p^p B_c^{\epsilon}}{(t_d-\tau)^{\delta}}\frac{r_{m\cdot u}^{\epsilon}}{r_m^{\delta}} \tag{8.34}$$

Q_u and Q_n is found to be approximated by

$$Q_u\fallingdotseq\left\{(\alpha_q)_{m\cdot u}/K_o^{1/p}\right\}\int_{\tau}^{t_d}r(t)dt$$
$$Q_n\fallingdotseq\left\{(\alpha_q)_{m\cdot n}/K_o^{1/p}\right\}\int_{\tau}^{t_d}r(t)dt$$

Hence, the following equation is obtained with setting p as 0.6 and ϵ as 0.4:

$$\gamma_Q=Q_u/Q_n\fallingdotseq\gamma_t\fallingdotseq\left\{0.6^{0.6}B_c^{0.4}/(t_d-\tau)^{2/3}\right\}\left(r_{m\cdot u}^{0.4}/r_m^{2/3}\right) \tag{8.35}$$

Equation (8.35) indicates that when τ becomes large, the error increases in the falling limbs of hydrograph. However, the absolute error is quite small, giving rise to no practical problem. Additionally, as $B_c^{0.4}$ is less than 10, γ_Q can be approximated with high accuracy as follows:

$$\gamma_Q=r_{m\cdot u}^{0.4}\Big/r_m^{2/3}=\left(r_{m\cdot u}\Big/r_m^{5/3}\right)^{0.4} \tag{8.36}$$

So far, the relative error γ_Q for each time interval has been computed between the discharge from the use of unit hydrograph method and the actual discharge. All derivations are based on the use of unit hydrograph with the unit rainfall (r_u, t_u), which is identified to make the peak of hydrograph coincide through the unit hydrograph method. The result reveals that the relative error hardly deviate from 1. Moreover, the discharge in the intervals (A), (B), and (D) is rather small in comparison with (C) near the peak discharge. Thus, the absolute error does not become large even if the deviation of γ_Q from 1 is considerably large. It is herein stressed that the elements of unit rainfall and unit hydrograph are identified to make the peak discharge coincide and such a concept can be extended to the analysis of a whole runoff process.

(b) The case of a large catchment $(t_u<t_c,\ t_d<t_c)$: The runoff mechanism for this case is schematically illustrated in **Fig. 8.7** and it is classified into the four time intervals, depending on α_t as follows:

$$(A)\ 0\le t<t_u\ ,\quad (B)\ t_u\le t<t_d\ ,\quad (C)\ t_d\le t<t_c\ ,\quad (D)\ t_c<t$$

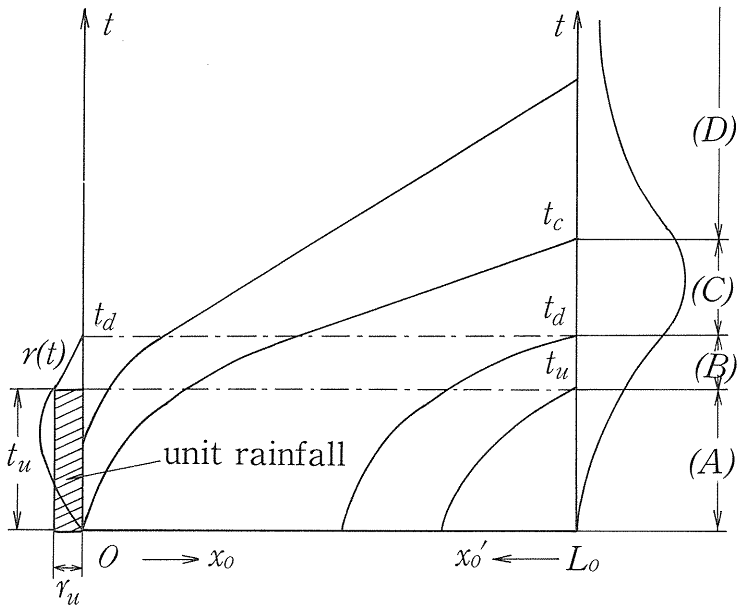

Fig. 8.7 Runoff mechanism in the case of $t_u<t_c$ and $t_d<t_c$

(A) $0 \leq t < t_u$: The following relation is obtained through eq.(5.6):

$$(\alpha_t)_{m\cdot u} = p^{P}(B_c')^{\epsilon} r_u^{\epsilon}, \quad (\alpha_t)_{m\cdot n} = p^{P}(B_c')^{\epsilon} r_m^{\epsilon}$$

As r_u and t_u are identified to make the peak discharge coincide and are given by eq.(6.14) as

$$\gamma_t = (\alpha_t)_{m\cdot u}/(\alpha_t)_{m\cdot n} = (t_{d\cdot u}/t_u)^{\epsilon}(r_{m\cdot u}/t_m)^{\epsilon} \tag{8.37}$$

On the other hand, the relarive error γ_Q of discharge is represented through the same procedure as in (A) of the previous section as follows:

$$\gamma_Q = \gamma_t(r_m'/r_m) = (t_{d\cdot u}/t_u)^{0.4}(r_{m\cdot u}/r_u)^{0.4}(r_m'/r_m) \tag{8.38}$$

where

$$r_m' = \int_0^{t_u} r(t)dt \Big/ \int_0^{t_u} dt, \quad r_m = \int_0^{t} r(t)dt \Big/ \int_0^{t} dt$$

(B) $t_u \leq t < t_c$: From eq.(5.6) and (5.7), the following relation is reduced:

$$(\alpha_t)_{m\cdot u} = B_c'(r_u t_u)^{\delta} \Big/ \left\{B_c' + \epsilon r_u^{\delta} t_u^{\delta+1}\right\}, \quad (\alpha_t)_{m\cdot n} = p^{P}(B_c')^{\epsilon} r_m^{\epsilon}$$

When using eq.(6.14), γ_t is expressed as

$$\gamma_t = \frac{(B_c')^{1-\epsilon}}{p^{P} r_m^{\epsilon}} \frac{(r_{m\cdot u} t_{d\cdot u})^{\delta}}{(B_c')^{1-\epsilon} + \epsilon t_u (r_{m\cdot u} t_{d\cdot u})^{\delta}} \fallingdotseq \frac{t_{d\cdot u}^{\epsilon} \; r_{m\cdot u}^{\delta}}{p^{P}(B_c')^{\epsilon} \; r_m^{\epsilon}} \tag{8.39}$$

$t^{\delta}_{d\cdot u}$ is approximated by $p^{P}(B_c')^{\epsilon}$ in a large catchment and γ_Q is represented in the same way as in (B) of the previous section as follows:

$$\gamma_Q \fallingdotseq \gamma_t \fallingdotseq r_{m\cdot u}^{2/3} \Big/ r_m^{0.4} \tag{8.40}$$

where

$$r_m = \int_0^t r(t)dt \Big/ \int_0^t dt$$

(C) $t_c \leq t < t_d$: Since α_t in this interval can be handled in the same manner as in 8. (1) (b), the following equation is obtained:

$$\gamma_Q = \gamma_t = \left(r_{m\cdot u} t_{d\cdot u} / r_m t_d\right)^{2/3} \tag{8.41}$$

where

$$r_m = \int_0^{t_d} r(t)dt \Big/ \int_0^{t_d} dt$$

(D) $t_d \leq t$: The following relations are formulated, corresponding to the starting time τ of propagation in eq. (5.7):

$$\begin{aligned} &for \quad t_d - \tau \geq t_u \ ; \quad (\alpha_t)_{m\cdot u} = B_c (r_u t_u)^{\delta} \Big/ \left\{B_c + \epsilon r_u^{\delta} t_u^{\delta+1}\right\} \\ &for \quad t_d - \tau < t_u \ ; \quad (\alpha_t)_{m\cdot u} = B_c \{r_u (t_u - \tau)\}^{\delta} \Big/ \left\{B_c + \epsilon r_u^{\delta} (t_u - \tau)^{\delta+1}\right\} \end{aligned}$$

and

$$(\alpha_t)_{m\cdot n} = B_c \{r_m (t_d - \tau)\}^{\delta} \Big/ \left\{B_c + \epsilon r_m^{\delta} (t_d - \tau)^{\delta+1}\right\}$$

By considering $B_c \gg \epsilon r_u^{\delta} t_u^{\delta+1}$, $\epsilon r_u^{\delta} (t_u - \tau)^{\delta+1}$, $\epsilon r_m^{\delta} (t_d - \tau)^{\delta+1}$ and using the above equations and eq.(6.14), the following relation is represented in the same approach as in (D) of the previous section:

$$\left.\begin{aligned} &for \quad t_d - \tau \geq t_u \ ; \quad \gamma_Q \fallingdotseq \gamma_t \fallingdotseq \left\{r_{m\cdot u} t_{d\cdot u} / r_m (t_d - \tau)\right\}^{2/3} \\ &for \quad t_d - \tau < t_u \ ; \quad \gamma_Q \fallingdotseq \gamma_t \fallingdotseq \left\{r_{m\cdot u} t_{d\cdot u} (t_u - \tau) / r_m t_u (t_d - \tau)\right\}^{2/3} \end{aligned}\right\} \tag{8.42}$$

The same conclusions can be drawn in the case of a large catchment as for a small catchment. The peak of hydrograph is matched by superposing the unit hydrograph with unit rainfall (r_u, t_u) in the unit hydrograph method. The relative error γ_Q for each time interval can easily be computed between the discharge from use of unit hydrograph approach and the actual discharge. The relative error does not largely deviate from 1 for each time interval. The absolute error becomes larger as the discharge is large, given the same value of γ_Q. It is concluded that a whole runoff process can be analyzed by using eq. (6.14) in terms of r_u and t_u, which gives the coincidence condition of peak discharge.

(3) Relationship between catchment scale and errors of unit hydrograph method

It is of interest to compare the errors between small and large catchments as mentioned in the previous two sections. For convenience, the subscripts s and l denote the quantity for small and large catchments, respectively. The discharges are given by eqs.(8.11) and(8.28) as

$$\gamma_{Q\cdot s} = \gamma_t + \nu(1-\gamma_t) \fallingdotseq \gamma_t = (r_{m\cdot u}/r_m)^{0.4}$$

$$\gamma_{Q\cdot l} = \gamma_t = (R_u/R_{td})^{2/3}$$

Thus, the relative error of a small catchment is a function of the ratio of rainfall intensity $r_{m \cdot u}$, which is a basic element to select the unit rainfall to average intensity r_m of the actual rainfall which causes the peak discharge. On the other hand, the relative error of a large catchment is given by a function of the ratio of total volume of unit rainfall to that of the actual rainfall. The following relation is commonly realized for actual flood events:

variation of $r_{m \cdot u}/r_m$ $>$ ***variation of*** R_u/R_{td}

It is because the total volume of rainfall generating flood does not change significantly in a catchment and the hourly rainfall intensity abruptly changes only around the peak. It is, therefore, obvious that the error of maximum discharge with the unit hydrograph method in a small catchment is larger than that in a large catchment.

As already mentioned in (a) and (b) of the previous sections, the relative error γ_Q of discharge for a whole hydrograph is determined by the catchment characteristics in a small catchment, while that by the duration time of rainfall as well as average rainfall intensity in a large catchment. Although the ratio of $r_{m \cdot u}$ to r_m is involved in both cases, this ratio deviated from 1 is obviously larger in a small catchment than that in a larger one from the definition of $r_{m \cdot u}$ and r_m. Therefore, the error related to a whole hydrograph in a small catchment is quite larger than that in a large catchment as for the case of maximum discharge.

9. CONCLUSIONS

To date, the unit hydrograph method has been well known to be one of the most effective methodologies for understanding the runoff mechanisms in mountainous catchments. Some efforts have long been made to clarify the issues involved in the application of unit hydrograph method and their outcome of researches can provide the answer to the relevant questions. Because of extreme complexity of rainfall behavior in mountainous catchments, most of conventional studies have dealt with the runoff phenomenon from a standpoint of black-box approaches that rely largely on the empirical facts. There is a great lack of universal information, even though their approaches can contribute to an engineering understanding of the rainfall-runoff process.

In the past, there were some arguments of whether or not the unit hydrograph method can be applicable in Japanese rivers with relatively small catchment areas. This is due to the fact that the difference of empirical facts was not fully understood, when the unit hydrograph method was used in the United States of America as well as in Japan.

In the present study, the behavior of rainfall is analyzed by a physically-based model that can facilitate the understanding of dynamic significance in the unit hydrograph theory. This study also aims to clarify the fundamental structures of unit rainfall and unit hydrograph, giving rise to the universal results in itself.

Relatively little research is being carried out to solve the relevant issues involved in the use of unit hydrograph method, e.g., the construction of synthetic unit hydrograph and application limit of unit hydrograph method to be discussed from the use of error concept. To remedy this situation, it is required to gain an insight into the physical meaning of unit hydrograph method. It is expected that the results of the present study, basing on the consistent aspect of hydraulic mechanisms, can give the answer to the above requests.

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